AMS 151 (Fall, 2009) Joe Mitchell

Applied Calculus I
Quiz # 1: Prerequisites Test – Solution Notes

1. Find all numbers x satisfying $2 - 1/x = 1/(1+x)$.

$$2 - \frac{1}{x} = \frac{1}{1+x}$$
$$\frac{2x-1}{x} = \frac{1}{1+x}$$
$$(2x-1)(1+x) = x$$
$$2x + 2x^2 - 1 - x = x$$
$$2x^2 = 1$$
$$x \in \{-\sqrt{1/2}, \sqrt{1/2}\}$$

2. $g(t)$ is an exponential function, and $g(0) = 10$, and $g(3) = 80$. Find an expression for $g(t)$.

Since $g(t)$ is exponential, we know that $g(t) = Ab^t$, for some A, b.
We know that $g(0) = 10$; thus, $A = 10$.
We know $g(3) = 80$, so $10b^3 = 80$, so $b^3 = 8$, so $b = 2$.
Thus, $g(t) = 10 \cdot 2^t$.

3. Solve this equation for t: $y = b + \frac{a(t-1)}{t+1}$.

$$yt + y = bt + b + at - a$$
$$y + a - b = (b + a - y)t$$
$$t = \frac{y + a - b}{a + b - y}$$

4. The inequality $2|2 - x| > \frac{2}{3}$ describes a subset of the real line (the subset of numbers x for which the inequality holds). Describe this subset.

We know that $|2 - x| > (1/3)$. Thus, $2 - x > (1/3)$ **OR** $2 - x < -(1/3)$. Thus, $x < 5/3$ **OR** $x > 7/3$. This can be written also as $x \in (-\infty, 5/3) \cup (7/3, \infty)$, a union of two halflines.

5. Find the equation of the line through point (2,3) that is perpendicular to the line $2y + x = 5$.

We desire a line L, with equation $y = mx + b$. Since L is to be perpendicular to the line $2y + x = 5$ (i.e., the line $y = (-1/2)x + 5/2$), we know that the slope $m = 2$ (the negative reciprocal of $(-1/2)$). Thus L has equation $y = 2x + b$.

Since point (2,3) lies on line L, we know that $3 = 2 \cdot 2 + b$, so $b = -1$.

Thus, line L is $y = 2x - 1$.

6. Find the set of real numbers satisfying $3x^2 + 6x < 45$.

We want values of x for which $x^2 + 2x - 15 < 0$. We can compute the two values of x where $x^2 + 2x - 15 = 0$: we can either factor the polynomial ($x^2 + 2x - 15 = (x - 3)(x + 5)$), or apply the quadratic formula to obtain the roots $x \in \{-5, 3\}$ where $x^2 + 2x - 15 = 0$.

Since $f(x) = x^2 + 2x - 15$ is concave upwards (the parabola "holds water", since the coefficient of x^2 is positive), we see that for values of x strictly between -5 and 3 the desired inequality holds: $x \in (-5, 3)$.

7. What is the area of the triangle bounded by the lines $3y - 3x = 0$, $y = -2x + 4$, and the y-axis.

Draw a picture of the 3 lines! We find the corners of the triangle are: (0,0), (0,4), and (4/3,4/3). (Lines $y = x$ and $y = -2x + 4$ meet at (4/3,4/3), as we get by solving the 2 equations in 2 unknowns.)

Let's treat the vertical side (from (0,0) to (0,4)) as the "base" (any of the 3 sides could be thought of as base), which has length 4. Then the "height" is 4/3.

The area is then $\frac{1}{2} \cdot 4 \cdot (4/3) = 8/3$.

8. Let $f(y) = \frac{|y-1|}{y+2}$ and let $g(x) = 2 - x$. Determine $f(g(2))$ and $g(f(-1))$.

We see that $g(2) = 0$. Then, $f(g(2)) = f(0) = \frac{|0-1|}{0+2} = \frac{1}{2}$.
We see that $f(-1) = 2$. Then, $g(f(-1)) = g(2) = 0$.

9. Find the equation of the line through the points (2,4) and (2,-9).

The line through these two points is a vertical line (infinite slope): $x = 2$.

10. Let $f(w) = w^2$ and $g(u) = 2 + \sqrt{u}$. Determine $f(g(y))$ and $g(f(x))$.

$f(g(y)) = (2 + \sqrt{y})^2$.
$g(f(x)) = 2 + \sqrt{x^2} = 2 + |x|$.

11. Calculate the area of a triangle with sides of length 9, 12, and 15.

Note that this is a right triangle (since $9^2 + 12^2 = 15^2$; this is just a 3-4-5 right triangle scaled up by a factor of 3).

Then the area is $\frac{1}{2} \cdot 12 \cdot 9 = 54$.

12. Find x satisfying $\log_{10}(x^2 + 1) - \log_{10}(x^2 - 3x + 1) = 1$.

$$\log_{10} \frac{x^2 + 1}{x^2 - 3x + 1} = 1$$

$$\frac{x^2 + 1}{x^2 - 3x + 1} = 10$$

$$9x^2 - 30x + 9 = 0$$

$$3x^2 - 10x + 3 = 0$$

By the quadratic formula, we get $x \in \{1/3, 3\}$.

13. Determine $\cos(11\pi/4)$.

$\cos \frac{11\pi}{4} = \cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

14. Triangle ABC is a right triangle, with right angle at the corner B. The hypotenuse has length 12, and edge AB has length 5.

(a). Find the area of triangle ABC.
(b). What is the $\sin(\angle BAC)$?
(c). What is $\tan(\angle BCA)$?

By the Pythagorean Theorem, we see that the length of edge BC is $\sqrt{12^2 - 5^2} = \sqrt{119}$.
(a). The area is $\frac{1}{2} \cdot 5 \cdot \sqrt{119}$
(b). $\sin \angle BAC = \frac{\sqrt{119}}{12}$
(c). $\tan \angle BCA = \frac{5}{\sqrt{119}}$

15. Find the (real) value of x for which $e^{e^{2x}} = e^{3^2}$.

$$e^{2x} = 3^2 = 9$$

$$2x = \ln 9$$

$$x = \frac{\ln 9}{2}$$