AMS 151 (Fall, 2009) Joe Mitchell

Applied Calculus I – Solution Notes
Quiz # 2

1. Solve for x: $400 \cdot 5^{2x} = 30 \cdot (1.22)^x$
   Taking the natural log of both sides, we get

$$\ln 400 + 2x \ln 5 = \ln 30 + x \ln 1.22$$

$$x(2\ln 5 - \ln 1.22) = \ln 30 - \ln 400$$

$$x = \frac{\ln 30 - \ln 400}{2\ln 5 - \ln 1.22} = \frac{\ln(3/40)}{\ln(25/1.22)}$$

2. Let $f(x) = x^3 - 27$ and $g(x) = e^{x-3}$.
   (a). Determine $g(f(x))$

$$g(f(x)) = g(x^3 - 27) = e^{(x^3-27)-3} = e^{x^3-30}$$

(b). Which function has a larger value as $x \to \infty$: $g(f(x))$ or $f(g(x))$?

Note that $f(g(x)) = (e^{x-3})^3 - 27 = e^{3x-9} - 27$, so $f(g(x))$ grows like e^{3x}, which does not grow as quickly as e^{x^3}, which is how $g(f(x))$ grows.

3. *The quantity of moisture in a frozen waffle decreases with time as it sits on the counter. Suppose that the moisture, $M(t)$, at time t minutes after being placed on the counter decreases according to the function $M(t) = Qe^{-kt}$. If 7% of the moisture is gone at the end of 15 minutes,*

(a). What percentage of the original moisture is present after 30 minutes?

We are told that $M(15) = (.93)M(0)$, which implies that

$$Qe^{-15k} = (.93)Q$$

$$-15k = \ln(.93)$$

$$k = \frac{-\ln(.93)}{15}$$

Now, we want to find

$$\frac{M(30)}{M(0)} \cdot 100 = \frac{Qe^{-30k}}{Q} \cdot 100 = 100e^{-30\frac{-\ln(.93)}{15}} = 100e^{2\ln(.93)} = 100(.93)^2$$

(b). How long will it take until the moisture is reduced to 50% of its original quantity?

We want to find t so that $M(t) = \frac{1}{2}M(0) = \frac{1}{2}Q$. Thus, we want t so that $Qe^{-kt} = \frac{1}{2}Q$, i.e., so that $e^{-kt} = \frac{1}{2}$. Thus, we want t so that $-kt = \ln\frac{1}{2}$, i.e.,

$$t = \frac{\ln\frac{1}{2}}{-k} = \frac{15\ln\frac{1}{2}}{\ln(.93)}$$

4. Plot $f(x) = 2\sin(x) - 1$. Be sure to mark important points on the axes!

The plot is a simple sine wave, oscillating between -3 and +1, with period 2π, since it has amplitude 2 and has been shifted downwards by 1.