AMS 151 (Fall, 2009) Joe Mitchell

Applied Calculus I
Quiz # 3 – Solution Notes

1. Evaluate the following expressions:
(a). $\sin(\cos^{-1}(\frac{5}{11})) =$

Draw a right triangle and let θ be one of the two angles that is not $\pi/2$. Then, $\theta = \cos^{-1}(\frac{5}{11})$ if the side adjacent to θ is of length 5 and the hypotenuse is of length 11. Then, the side opposite θ is of length $\sqrt{11^2 - 5^2} = \sqrt{96}$. Thus, $\sin(\cos^{-1}(\frac{5}{11})) = \sin\theta = \frac{\sqrt{96}}{11}$.

(b). $\tan(\cos^{-1}(\frac{\sqrt{3}}{2})) =$

Draw a right triangle and let θ be one of the two angles that is not $\pi/2$. Then, $\theta = \cos^{-1}(\frac{\sqrt{3}}{2})$ if the side adjacent to θ is of length $\sqrt{3}$ and the hypotenuse is of length 2. Then, the side opposite θ is of length $\sqrt{2^2 - 3} = 1$. (In fact, we know that $\theta = \pi/6$, 30 degrees.) Thus, $\tan(\cos^{-1}(\frac{\sqrt{3}}{2})) = \tan\theta = \frac{1}{\sqrt{3}}$.

(c). $\cos(\frac{40\pi}{3}) =$

Note that $\frac{40\pi}{3} = 13\pi + \frac{\pi}{3}$ (i.e., it is 6.5 revolutions, plus an additional angle $\pi/3$ (60 degrees). Thus, $\cos(\frac{40\pi}{3}) = \cos(4\pi/3) = -\frac{1}{2}$. (Draw a picture! The angle refers to a point in the third quadrant.)

2. A population of animals oscillates sinusoidally between a low of 200 on February 1 and a high of 1000 on May 1 (3 months later). Let t be the time in months since the start of the year (January 1). Find a formula for the population, $P(t)$, as a function of t.

Since $P(t)$ oscillates between 200 and 1000, we know that the amplitude is (1000-200)/2=400. The baseline of the oscillation occurs at $P = 200 + 400 = 600$; i.e., it is a sin/cos shifted upwards by 600.

Since the low value (min) occurs at $t = 1$ (Feb 1) and the first high (max) after that occurs at $t = 4$ (May 1), we know that half of the period is 4-1=3; thus, the period is 6 months.

Draw a picture! We can view $P(t)$ as a $-\cos$ function that is shifted right by 1, or as a sin function shifted right by 2.5, or as a cos function shifted right by 4, etc. Thus,

$$P(t) = 600 - 400\cos(\frac{2\pi}{6}(t-1))$$

or

$$P(t) = 600 + 400\sin(\frac{2\pi}{6}(t-2.5))$$

or

$$P(t) = 600 + 400\cos(\frac{2\pi}{6}(t-4))$$

3. Find the asymptotes for the following function:

$$y = \frac{2 - 19x + 7x^2}{2x^2 - 18}$$

(a). Vertical: The denominator, $2x^2 - 18 = 2(x-3)(x+3)$, vanishes at $x = 3$ and at $x = -3$, so these two values of x define the two vertical asymptotes.
(b). Horizontal:
We can rewrite (by dividing numerator and denominator by x^2)

$$y = \frac{2 - 19x + 7x^2}{2x^2 - 18} = \frac{\frac{2}{x^2} - \frac{19}{x} + 7}{2 - \frac{18}{x^2}}$$

from which we see the behavior as x go to infinity. As $x \to +\infty$, y goes up to 7/2. As $x \to -\infty$, y goes down to 7/2.
Thus, $y = 7/2$ is a horizontal asymptote.
(c). Sketch a plot of $y(x)$

Note that for $x = 3 + \epsilon$ (for tiny $\epsilon > 0$), y is positive, heading to $+\infty$. Note that for $x = 3 - \epsilon$ (for tiny $\epsilon > 0$), y is negative, heading to $-\infty$.

Note that for $x = -3 + \epsilon$ (for tiny $\epsilon > 0$), y is negative, heading to $-\infty$. Note that for $x = -3 - \epsilon$ (for tiny $\epsilon > 0$), y is positive, heading to $+\infty$.

As $x \to +\infty$, y goes up to 7/2. As $x \to -\infty$, y goes down to 7/2.

From these facts, we can sketch the plot. Check yourself by typing

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plot (2-19x+7x^2)/(2x^2-18)
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into wolframalpha.com, and you will see the plot.

(You will see that the function has a local minimum for a value of $x > 3$; when we study derivatives, we will see why.)

4. The displacement (in feet) of a car moving in a straight line is given by $s(t) = 2t^2 + 3$, where t is measured in seconds.

(a). Find the average velocity of the car over the time interval [9,13].

The average velocity over the interval is

$$\frac{s(13) - s(9)}{13 - 9} = \frac{(2 \cdot 13^2 + 3) - (2 \cdot 9^2 + 3)}{13 - 9} = 44 ft/sec$$

(b). Find the instantaneous velocity of the car when $t = 11$.

The instantaneous velocity at $t = 4$ is the limit as ϵ goes to zero of

$$\frac{s(11 + \epsilon) - s(11)}{11 + \epsilon - 11}$$

which is the derivative of $s(t)$ evaluated at $t = 11$: $s'(t) = 4t$, so $s'(11) = 44$ ft/sec.