1. Evaluate the following expressions:
   (a). \( \sin(\cos^{-1}\left(\frac{5}{11}\right)) = \)

   Draw a right triangle and let \( \theta \) be one of the two angles that is not \( \pi/2 \). Then, \( \theta = \cos^{-1}\left(\frac{5}{11}\right) \) if the side adjacent to \( \theta \) is of length 5 and the hypotenuse is of length 11. Then, the side opposite \( \theta \) is of length \( \sqrt{11^2 - 5^2} = \sqrt{96} \). Thus, \( \sin(\cos^{-1}\left(\frac{5}{11}\right)) = \sin \theta = \frac{\sqrt{96}}{11} \).

   (b). \( \tan(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)) = \)

   Draw a right triangle and let \( \theta \) be one of the two angles that is not \( \pi/2 \). Then, \( \theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \) if the side adjacent to \( \theta \) is of length \( \sqrt{3} \) and the hypotenuse is of length 2. Then, the side opposite \( \theta \) is of length \( \sqrt{2^2 - 3} = 1 \). (In fact, we know that \( \theta = \pi/6 \), 30 degrees.) Thus, \( \tan(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)) = \tan \theta = \frac{1}{\sqrt{3}} \).

   (c). \( \cos\left(\frac{40\pi}{3}\right) = \)

   Note that \( \frac{40\pi}{3} = 13\pi + \frac{\pi}{3} \) (i.e., it is 6.5 revolutions, plus an additional angle \( \pi/3 \) (60 degrees). Thus, \( \cos\left(\frac{40\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2} \). (Draw a picture! The angle refers to a point in the third quadrant.)

2. A population of animals oscillates sinusoidally between a low of 200 on February 1 and a high of 1000 on May 1 (3 months later). Let \( t \) be the time in months since the start of the year (January 1). Find a formula for the population, \( P(t) \), as a function of \( t \).

   Since \( P(t) \) oscillates between 200 and 1000, we know that the amplitude is \((1000-200)/2=400\). The baseline of the oscillation occurs at \( P = 200 + 400 = 600 \); i.e., it is a sin/cos shifted upwards by 600.

   Since the low value (min) occurs at \( t = 1 \) (Feb 1) and the first high (max) after that occurs at \( t = 4 \) (May 1), we know that half of the period is 4-1=3; thus, the period is 6 months.

   Draw a picture! We can view \( P(t) \) as a \(-\cos\) function that is shifted right by 1, or as a \(\sin\) function shifted right by 2.5, or as a \(\cos\) function shifted right by 4, etc. Thus,

   \[
   P(t) = 600 - 400 \cos\left(\frac{2\pi}{6}(t - 1)\right)
   \]

   or

   \[
   P(t) = 600 + 400 \sin\left(\frac{2\pi}{6}(t - 2.5)\right)
   \]

   or

   \[
   P(t) = 600 + 400 \cos\left(\frac{2\pi}{6}(t - 4)\right)
   \]

3. Find the asymptotes for the following function:

   \[
   y = \frac{2 - 19x + 7x^2}{2x^2 - 18}
   \]

   (a). Vertical: The denominator, \( 2x^2 - 18 = 2(x - 3)(x + 3) \), vanishes at \( x = 3 \) and at \( x = -3 \), so these two values of \( x \) define the two vertical asymptotes.

   (b). Horizontal:

   We can rewrite (by dividing numerator and denominator by \( x^2 \))

   \[
   y = \frac{2 - 19x + 7x^2}{2x^2 - 18} = \frac{\frac{2}{x^2} - \frac{19}{x} + 7}{2 - \frac{18}{x^2}}
   \]

   from which we see the behavior as \( x \) go to infinity. As \( x \to +\infty \), \( y \) goes up to 7/2. As \( x \to -\infty \), \( y \) goes down to 7/2.

   Thus, \( y = 7/2 \) is a horizontal asymptote.

   (c). Sketch a plot of \( y(x) \)

   Note that for \( x = 3 + \epsilon \) (for tiny \( \epsilon > 0 \), \( y \) is positive, heading to \(+\infty\). Note that for \( x = 3 - \epsilon \) (for tiny \( \epsilon > 0 \), \( y \) is negative, heading to \(-\infty\).
Note that for \( x = -3 + \epsilon \) (for tiny \( \epsilon > 0 \)), \( y \) is negative, heading to \( -\infty \). Note that for \( x = -3 - \epsilon \) (for tiny \( \epsilon > 0 \)), \( y \) is positive, heading to \( +\infty \).

As \( x \to +\infty \), \( y \) goes up to 7/2. As \( x \to -\infty \), \( y \) goes down to 7/2.

From these facts, we can sketch the plot. Check yourself by typing

\[ \text{plot} \ (2-19x + 7x^2)/(2x^2-18) \]

into wolframalpha.com, and you will see the plot.

(You will see that the function has a local minimum for a value of \( x > 3 \); when we study derivatives, we will see why.)

4. The displacement (in feet) of a car moving in a straight line is given by \( s(t) = 2t^2 + 3 \), where \( t \) is measured in seconds.

   (a). Find the average velocity of the car over the time interval \([9, 13]\).

   The average velocity over the interval is
   \[
   \frac{s(13) - s(9)}{13 - 9} = \frac{(2 \cdot 13^2 + 3) - (2 \cdot 9^2 + 3)}{13 - 9} = 44 \text{ ft/sec}
   \]

   (b). Find the instantaneous velocity of the car when \( t = 11 \).

   The instantaneous velocity at \( t = 4 \) is the limit as \( \epsilon \) goes to zero of
   \[
   \frac{s(11 + \epsilon) - s(11)}{11 + \epsilon - 11}
   \]

   which is the derivative of \( s(t) \) evaluated at \( t = 11 \): \( s'(t) = 4t \), so \( s'(11) = 44 \text{ ft/sec} \).