Applied Calculus I
Quiz # 4, Thursday, October 22 – Solution Notes

1. Find the equation of the tangent line to the curve $y = 2\sqrt{x} + 1$ $(x \geq 0)$ at the point (4,5).

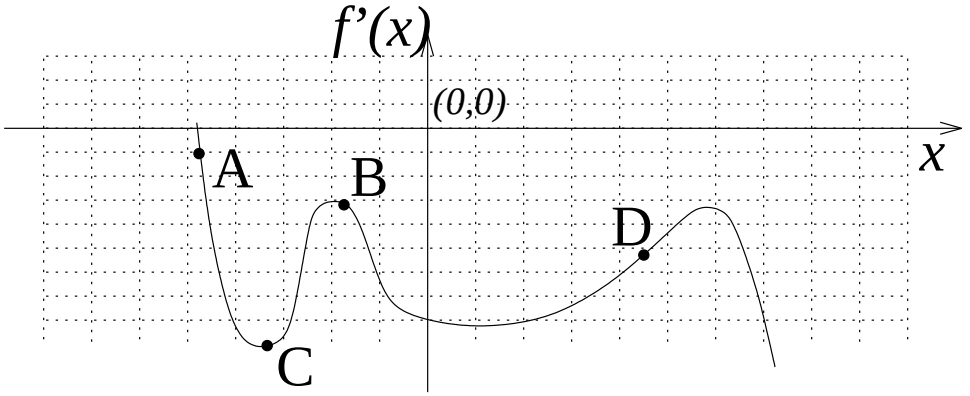
Since the slope is given by the derivative function, $y'(x) = 2 \cdot (1/2)x^{-1/2} = \frac{1}{\sqrt{x}}$, we know that the slope of the tangent line at (4,5) is $y'(4) = 1/2$. Thus, the equation of the tangent line at (4,5) has the form $y = (1/2)x + b$, and we can find b using the fact that the line must pass through the point (4,5): $5 = (1/2) \cdot 4 + b$, implying that $b = 3$. Thus, the equation of the tangent line at (4,5) is $y = (1/2)x + 3$.

2. If $h(b) = xb^3 + b^2 - \frac{x^2}{b}$, find $h'(1)$. Also find $h''(x)$.

$h'(b) = x \cdot 3b^2 + 2b - x^2 \cdot (-1)b^{-2} = 3xb^2 + 2b + \frac{x^2}{b^2}$. Taking the second derivative, we get $h''(b) = 6xb + 2 + x^2 \cdot (-2)b^{-3} = 6xb + 2 - \frac{2x^2}{b^3}$.

Thus, $h'(1) = 3x + 2 + x^2$. Also, $h''(x) = 6x^2 + 2 - \frac{2}{x}$.

3. The graph of $f'(x)$ is shown below. At which of the marked points is (a). $f(x)$ the least? (b). $f(x)$ the greatest? (c). $f'(x)$ the least? (d). $f'(x)$ the greatest? (e). $f''(x)$ the least? (f). $f''(x)$ the greatest?



(a), (b). Since $f'(x) < 0$ over the interval of x-coordinates of interest (spanning all four points), we know that $f(x)$ is *decreasing* over the interval. Thus, the least value of $f(x)$ occurs at D and the greatest occurs at A.

(c), (d). Among the four points marked, $f'(x)$ is greatest at A and is least at C.

(e), (f). Among the four points marked, the slope, $f''(x)$, of $f'(x)$ is greatest at D (where the slope is positive; the slope is about zero at C and is negative at A and B) and is least at A (where the slope is negative, as it is also at B, but it is more negative at A). Thus, $f''(x)$ is greatest at D and is least at A.

4. At a time t after it is thrown up in the air, a tomato is at a height of $f(t) = -4t^2 + 8t + 3$ meters.

(a). What is the velocity of the tomato at time $t = 2$? Is the tomato going up or going down at time $t = 2$?

The velocity at time t is given by $f'(t) = -8t + 8$. Thus, the velocity at time $t = 2$ is $f'(2) = -8 \cdot 2 + 8 = -8$. Since the velocity is negative, the height is decreasing at time $t = 2$; i.e., the tomota is going down.

(b). Find the acceleration of the tomato at time $t = 2$.

The acceleration at time t is given by the second derivative, $f''(t) = -8$. (It is constant, pointed downwards.) Thus, the acceleration at time $t = 2$ is $f''(2) = -8$.

(c). How high does the tomato go?

The tomota reaches it highest point when the velocity is 0, i.e., when $f'(t) = 0$, or $-8t + 8 = 0$, which means that $t = 1$. The height at time $t = 1$ is $f(1) = -4 \cdot 1^2 + 8 \cdot 1 + 3 = 7$ meters.