Operations Research II: Stochastic Models
Homework Set # 2

Due at the beginning of class on Wednesday, February 7, 2007. Reminder: Show your reasoning!

Recommended Reading: Ross, Chapter 3, Sections 3.1–3.5. (also, problem (3) is related to Section 3.6.4, but you may find this section challenging to read – concentrate on Case 1).

Note: All problem numbers refer to the text, Ross, Ninth Edition. (If you use an earlier edition of the text, you will have to determine the correct problems to do.)

(1). (16 points) # 5, Chapter 3: An urn contains three white, six red, and five black balls. Six of these balls are randomly selected from the urn. Let $X$ and $Y$ denote, respectively, the number of white and black balls selected. Compute the conditional probability mass function of $X$ given that $Y = 3$. Also compute $E(X \mid Y = 1)$.

(2). (16 points) # 15, Chapter 3: The joint density of $X$ and $Y$ is given by
\[
f(x, y) = \begin{cases} 
eg x-y & \text{if } 0 < x < y \text{ and } 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}
\]
Compute $E(X^2 \mid Y = y)$.

(3). (16 points) # 27, Chapter 3: A coin that comes up heads with probability $p$ is continually flipped until the pattern T, T, H appears. (That is, you stop flipping when the most recent flip lands heads, and the two immediately preceding it land tails.) Let $X$ denote the number of flips made, and find $E(X)$. (Hint: Condition! First condition on the first flip, then, you may also want to condition on second flip (in some cases). A geometric random variable is also relevant in the derivation, since a Geometric($p$) counts how many flips until the first Heads.)

(4). (16 points) # 41, Chapter 3: A rat is trapped in a maze. Initially it has to choose one of two directions. If it goes to the right, then it will wander around in the maze for three minutes and will then return to its initial position. If it goes to the left, then with probability 1/3 it will depart the maze after two minutes of traveling, and with probability 2/3 it will return to its initial position after five minutes of traveling. Assuming that the rat is at all times equally likely to go to the left or the right, what is the expected number of minutes that it will be trapped in the maze?

(5). (18 points) # 44, Chapter 3: The number of customers entering a store on a given day is Poisson distributed with mean $\lambda = 10$. The amount of money spent by a customer is uniformly distributed over $(0,100)$. Find the mean and variance of the amount of money that the store takes in on a given day.

(6). (18 points) # 57, Chapter 3: The number of storms in the upcoming rainy season is Poisson distributed but with a parameter value that is uniformly distributed over $(0,5)$. That is, $\Lambda$ is uniformly distributed over $(0,5)$, and given that $\Lambda = \lambda$, the number of storms is Poisson with mean $\lambda$. Find the probability there are at least three storms this season.