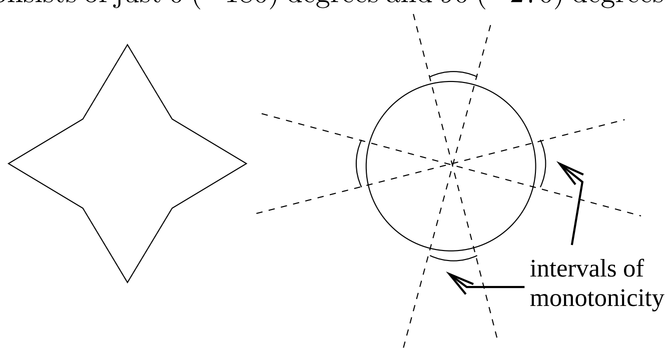
COMPUTATIONAL GEOMETRY
Homework Set # 3 – Solution Notes

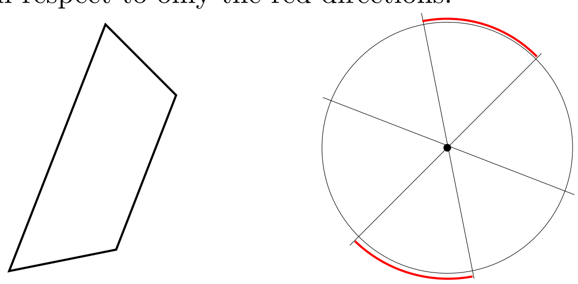
(1). *Give an example of a monotone simple polygon for which the set of directions d with respect to which it is monotone consists of at least two distinct double-cones of directions. (A* double-cone *of directions consists of an interval of angles, and the interval of their opposites; e.g., a double-cone may consist of angles (in degrees) in the intervals (10,45) and (190, 225), or of angles in the intervals (-10,10) and (170,190).) Show the set of directions with respect to which your polygon is monotone (e.g., highlight arcs on a circle to show the set of directions/angles).*

There are many possible examples. One class of examples is based on taking a convex polygon and "denting inward" each of its sides. See the example below. Another example comes from a "cross" or "plus" sign, "+", for which the set of directions with respect to which the (rectilinear) polygon is monotone consists of just 0 (=180) degrees and 90 (=270) degrees (two "point" intervals).

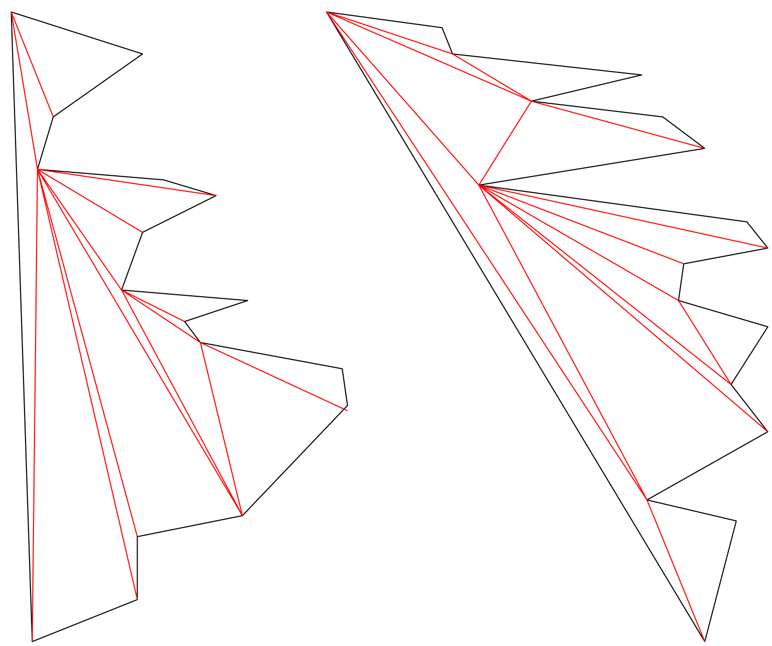


(2). *Give an example of a monotone mountain for which the set of directions d for which it is a monotone mountain is* different from *the set of all directions for which it is a monotone polygon.*

A convex quadrilateral (4-sided polygon) suffices: see the figure below, in which I show in the diagram to the right the set of directions (in red) that correspond to the polygon being a monotone mountain. Since the polygon is convex, it is *monotone* with respect to *all* directions; however, it is a monotone mountain with respect to only the red directions.

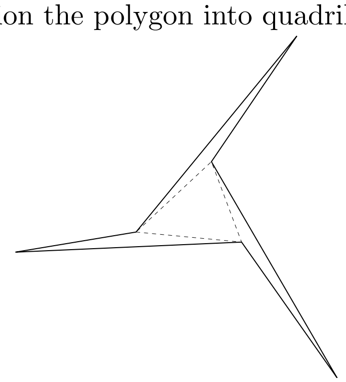


(3). *For the monotone mountains shown below, show the (unique) triangulation that is given by the algorithm we presented in class for triangulating monotone mountains (in which we always choose the* highest *strictly convex vertex (other than the endpoints of the base) to be the ear tip). (Use a ruler or straightedge; some of the points are nearly colinear! Larger images are on the web site.)*



(4). *O'Rourke, problems 5 and 6, section 2.3.4, page 55. (A "proof" that you can always do it should give a simple recipe for how to do it; for a "proof" that you cannot do it, you only need to show a counterexample and say briefly why it is a counterexample.) NOTE: O'Rourke fails to mention in problem 5 that you should assume that the number of vertices is $n \geq 6$. (Since, otherwise, a nonconvex quadrilateral is a trivial counterexample!)*

The polygon below is a counterexample to both problems 5 and 6: There are only 3 possible diagonals, and none of them partition the polygon into quadrilaterals (convex or nonconvex).



(5). *For each of the simple polygons P below, do the following:*

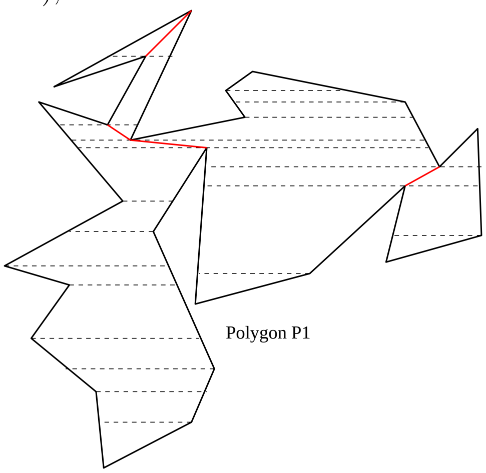
(a). Show the decomposition into monotone polygons given by the algorithm of Section 2.2. Show the resulting diagonals in RED.

(b). Show the decomposition into monotone mountains given by the algorithm of Section 2.3. Show the resulting diagonals in BLUE. (You may use a separate copy from the picture for (a).)

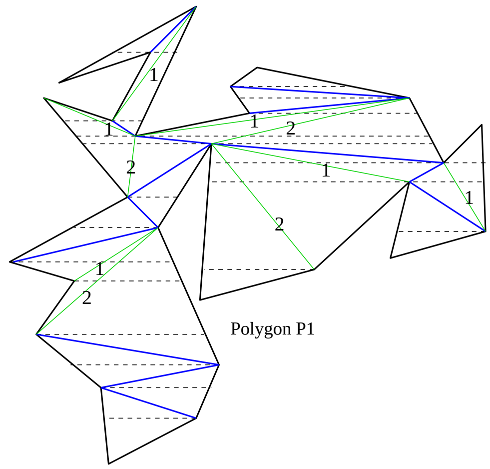
(c). Show the complete triangulation given by the triangulation algorithm of Section 2.3 (based on the monotone mountain partition), according to the specific rules we gave in class, as in problem (3) above. Show the diagonals in GREEN that partition each monotone mountain into triangles. Label the diagonals in the order that they are discovered by the algorithm (Algorithm 2.2, page 53, made specific, as we did in class).

I show the figures below. You can see the color of the segments (red, blue, green) in the electronic versions that are posted on the web.

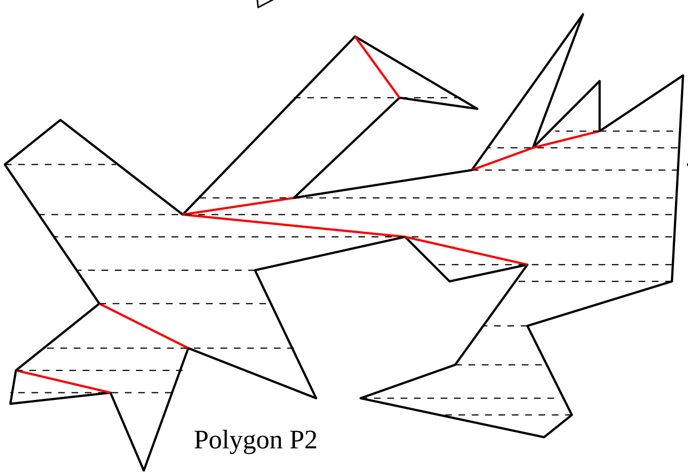
I number the green diagonals with “1”, “2”, “3”, etc, to indicate the order in which they will be found in the monotone mountain triangulation of each separate piece (hence, there are several “1’s”), assuming that we triangulate from “top to bottom” (in the search for convex vertices that define ears), as we did in class.



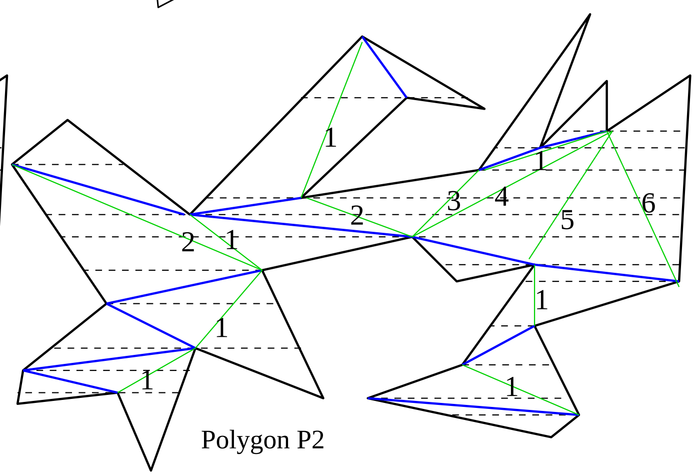
Polygon P1



Polygon P1



Polygon P2



Polygon P2