AMS 345/CSE 355 (Spring, 2008) Joe Mitchell

COMPUTATIONAL GEOMETRY
Homework Set # 3

Due at the beginning of class on Wednesday, March 5, 2008.

Recommended Reading: O'Rourke, Chapter 2 (sections 2.1–2.4).

Reminder: In all of the exercises, be sure to give at least a brief explanation or justification for each claim that you make.

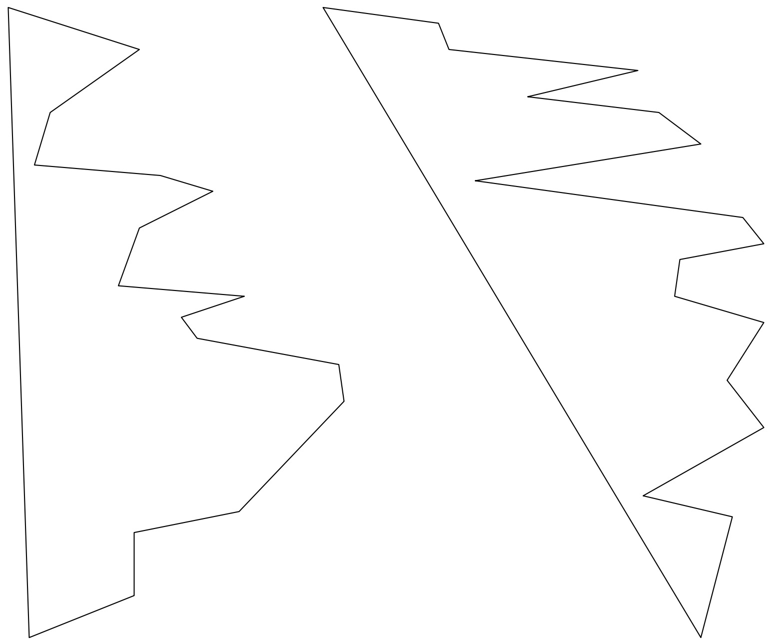
(1). [20 points] Give an example of a monotone simple polygon for which the set of directions d with respect to which it is monotone consists of at least two distinct double-cones of directions. (A *double-cone* of directions consists of an interval of angles, and the interval of their opposites; e.g., a double-cone may consist of angles (in degrees) in the intervals (10,45) and (190, 225), or of angles in the intervals (-10,10) and (170,190).) Show the set of directions with respect to which your polygon is monotone (e.g., highlight arcs on a circle to show the set of directions/angles).

(Optional question to provoke thought: For an n-gon, what is the maximum number of intervals (in terms of n) of directions of monotonicity? I know that it can grow like $\Omega(n)$, but can you get a tight exact bound?)

(2). [14 points] Give an example of a monotone mountain for which the set of directions d for which it is a monotone mountain is *different from* the set of all directions for which it is a monotone polygon.

(Optional question to provoke extra thought: Is it possible for the set of directions with respect to which P is a monotone *mountain* to consist of at least two distinct cones of directions?)

(3). [16 points] For the monotone mountains shown below, show the (unique) triangulation that is given by the algorithm we presented in class for triangulating monotone mountains (in which we always choose the *highest* strictly convex vertex (other than the endpoints of the base) to be the ear tip). (Use a ruler or straightedge; some of the points are nearly colinear! Larger images are on the web site.)



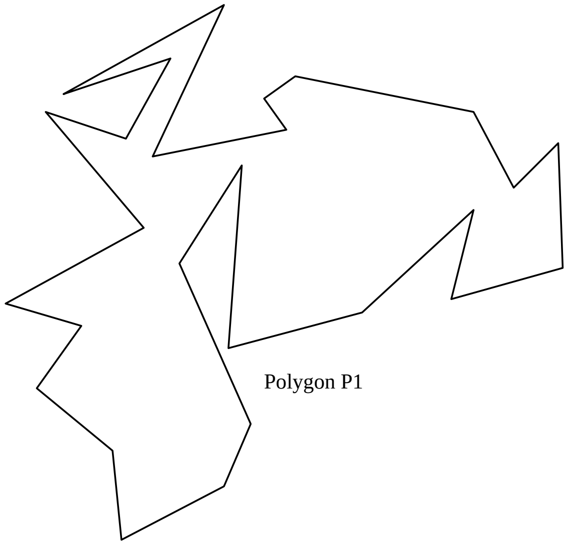
(4). [20 points] O'Rourke, problems 5 and 6, section 2.3.4, page 55. (A "proof" that you can always do it should give a simple recipe for how to do it; for a "proof" that you cannot do it, you only need to show a counterexample and say briefly why it is a counterexample.) NOTE: O'Rourke fails to mention in problem 5 that you should assume that the number of vertices is $n \geq 6$. (Since, otherwise, a nonconvex quadrilateral is a trivial counterexample!)

(5). [30 points] For each of the simple polygons P below, do the following:

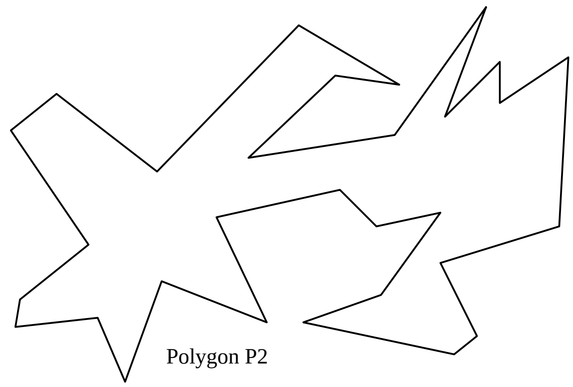
(a). Show the decomposition into monotone polygons given by the algorithm of Section 2.2. Show the resulting diagonals in RED.

(b). Show the decomposition into monotone mountains given by the algorithm of Section 2.3. Show the resulting diagonals in BLUE. (You may use a separate copy from the picture for (a).)

(c). Show the complete triangulation given by the triangulation algorithm of Section 2.3 (based on the monotone mountain partition), according to the specific rules we gave in class, as in problem (3) above. Show the diagonals in GREEN that partition each monotone mountain into triangles. Within each monotone mountain, label the diagonals in the order that they are discovered by the algorithm (Algorithm 2.2, page 53, made specific, as we did in class).



Polygon P1



Polygon P2