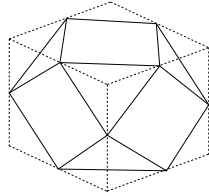


COMPUTATIONAL GEOMETRY

Homework Set # 5 – Solution Notes

(1). *O'Rourke, problem 4, section 4.1.6, page 108.* The cuboctahedron has $F = 8 + 6 = 14$ faces (one equilateral triangle face per corner of the original cube, plus one diamond face per original face of the cube). It has $E = 6 \cdot 4 = 24$ edges. It has $V = 12$ vertices. Thus, we can easily verify Euler's formula: $F - E + V = 14 - 24 + 12 = 2$.

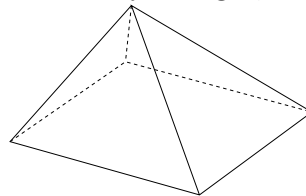


(2). (a). Assume there exists a 7-edge polyhedron. We will arrive at a contradiction.

Let V be the number of vertices, $E = 7$ the number of edges, and F the number of faces of the polyhedron. We know that each vertex of any polyhedron has degree at least 3 (this follows from the definition, since a vertex must lie at the intersection of 3 or more faces (it takes 3 planes to determine a point)). Thus, the sum of the vertex degrees, which equals $2E = 14$, is at least $3V$: $14 \geq 3V$ (which implies that the integer V must be at most 4). But the inequality $E \leq 3V - 6$ (which holds for planar graphs) implies that $7 \leq 3V - 6$, so $13 \leq 3V$, which contradicts the fact that $14 \geq 3V$. Thus, a polyhedron with $E = 7$ cannot exist.

(b). *Is there a polyhedron with exactly 8 edges? Justify! (draw one, or argue that none exists)* Yes, there are polyhedra with exactly 8 edges; as shown below (left), a square-based pyramid is such an example. In fact, if you think about it you should be able to argue that *any* 8-edge polyhedron has this structure: 4 faces that are triangles and one face that is a quadrilateral.

(c). *Is there a polyhedron with exactly 9 edges? Justify! (draw one, or argue that none exists)* Yes, there are polyhedra with exactly 9 edges; as shown below (middle and right).



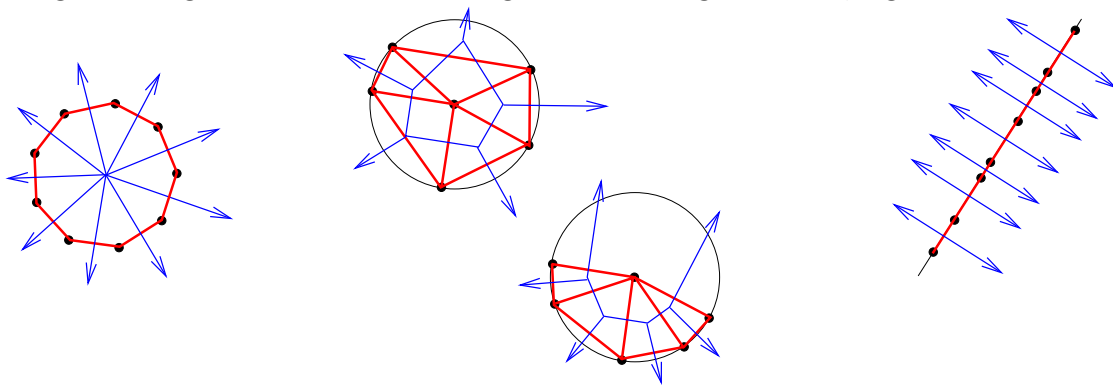
(3). *O'Rourke, problem 1, section 4.4.4, page 149.* We are given a vertex v and a winged edge data structure. Vertex v points to an edge, e , that is incident on v . Let $E_1 = e$ be the first edge that we print. Let $i = 1$. We look at the v_0 pointer ($v_0(E_i)$) for E_i . If $v_0 = v$ then we set $E_{i+1} = e_0^+(E_i)$; otherwise, we set $E_{i+1} = e_1^+(E_i)$. In either case, we get E_{i+1} as the edge that is next in the clockwise order about v . We then set $i = i + 1$ and continue around v until we get back to the original edge e (i.e., until $E_{i+1} = e$).

(4). Describe in words and draw figures for the Voronoi diagram (in blue) and Delaunay diagram (in red) for each of the following sets of points.

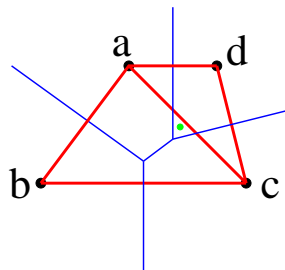
(a). *O'Rourke, problem 1, section 5.3.3, page 164.* The Voronoi diagram of the vertices of a regular n -gon consists of one Voronoi vertex (at the center of the polygon), with n equally-spaced rays (Voronoi edges) emanating from it, each being a perpendicular bisector of an edge of the n -gon. The Delaunay diagram consists exactly of the n edges of the n -gon, forming a cycle of n edges. See the figure below, left.

(b). *5 points all lying on a common circle (at arbitrary positions on the circle), plus a point at the center of the circle* There are two cases, depending on whether the center point lies on the convex hull of the 6 sites or not. Both cases are shown in the middle of the figure below. In both cases, the Delaunay diagram is a triangulation consisting of 5 spokes from the center site to the others, as well as other edges of the convex hull.

(c). *8 points all lying on a common line (at arbitrary positions on the line)* The Voronoi diagram of a set of n collinear points on line L in the plane consists of zero Voronoi vertices and $n - 1$ Voronoi edges, each of which is an infinite line perpendicular to the line L , bisecting the interval between two consecutive point sites along L . The Delaunay diagram consists of $n - 1$ edges linking consecutive sites along L . See the figure below, right.



(5). *O'Rourke, problem 6, section 5.3.3, page 165.* (a). Many examples exist (including the figure 5.6 of the text and the example in problem (6) below). Here is a simple, 2-triangle example, showing that the green point inside triangle (a, b, c) has vertex d as its nearest neighbor.



(b). A Delaunay triangulation is Pitteway if and only if each interior (non-convex-hull) Delaunay edge intersects exactly one Voronoi edge (namely, the one that is its dual). The proof of this fact requires a bit of care (case analysis), but is not particularly difficult. (See the textbook of Okabe et al.)

(6). Let S be the set of points $\{(-1,-2), (2,2), (5,6), (8,6), (9,2), (12,2), (12,-2), (8,-2), (5,-2)\}$. Construct and draw the Delaunay diagram (in red) and the Voronoi diagram (in blue). (We assume the usual Euclidean metric.)

