

COMPUTATIONAL GEOMETRY

Homework Set # 7

Due at the beginning of class on Monday, May 5, 2008.

(1). [20 points] For each of the following sign vectors, draw an arrangement of 5 distinct lines, $\{\ell_1, \dots, \ell_5\}$, that has a face (either a 2-face (cell), a 1-face (edge), or a 0-face (vertex)) with the corresponding sign vector (where “+” means lying strictly above a line, “-” means strictly below, and “0” means on the line). Clearly label each line and highlight the face.

Give the dimension of each such face. Also, say for each case whether the arrangement is *simple* or not. (You should make your example simple if it is possible to do so.)

(a). $(+, +, +, -, -)$; (b). $(0, +, 0, -, 0)$; (c). $(0, -, +, -, +)$; (d). $(-, -, -, +, 0)$; (e). $(0, 0, 0, -, 0)$

(Optional question for extra thought: Is it possible to find *one* set of 5 lines whose arrangement contains faces with all five sign vectors? Why or why not?)

(2). [20 points] Let L be a set of n distinct lines in the plane, no two of which are parallel. Assume that exactly k ($k \leq n$) of the lines (call this set L_0) all pass through a common point, p_0 . Assume that no three of the other $n - k$ lines (in the set $L_1 = L \setminus L_0$) pass through a common point, and no line of L_1 passes through p_0 . Assume also that no line of L_0 passes through an intersection point between two lines of L_1 . Derive formulas for the number of vertices, edges, and faces (cells) in the arrangement, $\mathcal{A}(L)$, of the lines L . Your formulas for v , e , and f will depend on n and k .

(3). [20 points] O’Rourke, problem 3(a), section 6.5.3, page 205.

Draw the lines that are the polar duals of the points $(2, -1)$, $(2, 3)$, $(1, 0)$, $(-0.5, 0.25)$, $(0.25, -0.5)$.

(4). [15 points] Describe what the dual is for a *right* triangle, Δpqr , with pq vertical (parallel to the y -axis) and qr horizontal (parallel to the x -axis). (The triangle is considered to be a closed region, consisting of all points inside or on the boundary of the triangle. Use the standard duality transform of the text (not polar duality).)

As an example, draw a picture to represent the dual of the triangle determined by the points $p = (3/2, 1)$, $q = (3/2, 3)$, and $r = (4, 3)$. (Label your axes and make it clear what the important points are.)

(5). [25 points] Let S be the set of points $\{p_1, \dots, p_8\} = \{(-4, -1), (-4, 5), (-1, -1), (-1, 3), (4, -1), (4, 3), (6, 6), (7, 3)\}$.

(a). Draw the arrangement of the duals to these points. (It is very useful to use graph paper or a computer drawing package for this. Make sure that you label the coordinate axes with important values of the coordinates.) Label the lines, letting ℓ_i denote the dual of point p_i .

(b). Let L be the y -axis. Highlight the edges of the cells of the zone of L (color them red). How many are there? How does this number compare to the bound given by Theorem 6.2.2?

(c). Which lines ℓ_i correspond to points p_i that are vertices on the upper convex hull of S ? Which lines ℓ_i correspond to points p_i that are vertices on the lower convex hull of S ? What do you observe about the connection between the convex hull of S and the arrangement of dual lines?

(d). Suppose that points $p_1 - p_4$ are “red” and $p_5 - p_8$ are “blue”. Give a ham sandwich cut of the points.

Extra Credit (mostly for fun) [10 points]:

An old parchment describes the location of buried treasure:

“On the island there are only two trees, A and B, and the remains of a gallows. Start at the gallows and count the steps required to walk in a straight line to tree A. At the tree turn 90 degrees to the left and then walk forward the same number of steps. At the point where you stop, drive a spike into the ground. Now return to the gallows and walk in a straight line, counting your steps, to tree B. When you reach the tree, turn 90 degrees to the right and take the same number of steps forward, placing another spike at the point where you stop. Dig at the point exactly halfway between the spikes and you will find the treasure.”

However, our hero when he gets to the island finds the gallows missing. Is there any way he can still get to the treasure? (Justify your answer.)