

COMPUTATIONAL GEOMETRY

Homework Set # 3

Due at the beginning of class on Thursday, October 18, 2007 *Reminder: Show your reasoning!*

Recommended Reading: BKOS: Chapter 3; O'Rourke, Chapters 1 and 2.

DO ANY 4 OF THE FOLLOWING 5 PROBLEMS.

(1). [25 points] Let P be a polygon having exactly one hole, H . (You may assume that the outer boundary of P consists of a (simple) cycle having n vertices, and that the single hole H is described by a (simple) cycle of m vertices, strictly contained within the outer boundary. For simplicity, you may assume that no three vertices of P are colinear.)

Suppose that we want to “bridge” H to the outer boundary of P by finding a chord that joins the boundary of H to the outer boundary of P . Argue that such a chord exists that has one endpoint at a vertex of H and one endpoint at a vertex of the outer boundary of P . Further, give an $O(n + m)$ time algorithm to find one such bridge, without first having to perform any triangulation. (Would your algorithm be easy to implement? Try to be fairly precise in stating the algorithm. You may use primitives that O'Rourke discusses in his book.) Give some brief justification of why your algorithm is correct.

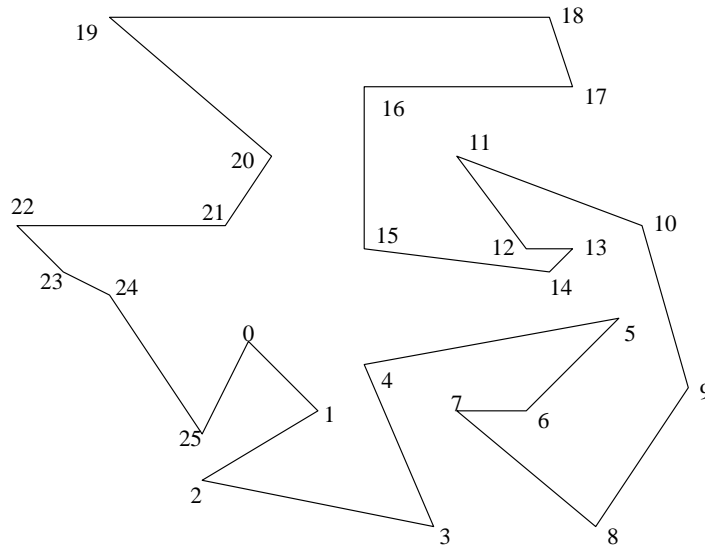
(2). [25 points] Chazelle has shown that one can determine if a given list of n vertices constitutes a simple polygon in time $O(n)$. Using this as a “black box”, explain in detail how you would check if a simple polygon P intersects a simple polygon Q in time $O(|P| + |Q|)$.

(3). [25 points]

- (a). Problem 3.2, page 60, BKOS.
- (b). Problem 3.3, page 60, BKOS.
- (c). Problem 3.4, page 60, BKOS.

(4). [25 points] For the simple polygon below, give the order in which diagonals are output when `Triangulate` is executed on it (exactly as is done in Table 1.2 for the polygon shown in Figure 1.27 of O'Rourke). The coordinates of the points (in order 0–25) are given by: (12,12), (15,9), (10,6), (20,4), (17,11), (28,13), (24,9), (21,9), (27,4), (31,10), (29,17), (21,20), (24,16), (26,16), (25,15), (17,16), (17,23), (26,23), (25,26), (6,26), (13,20), (11,17), (2,17), (4,15), (6,14), (10,8).

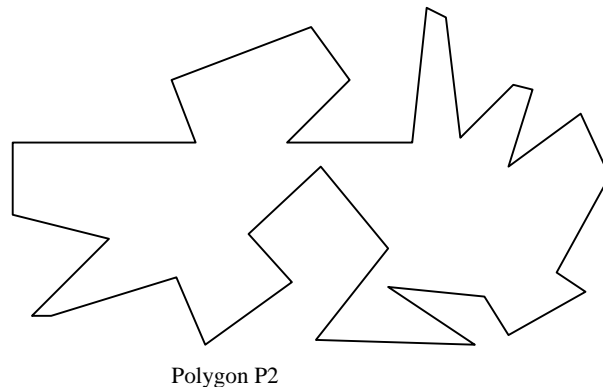
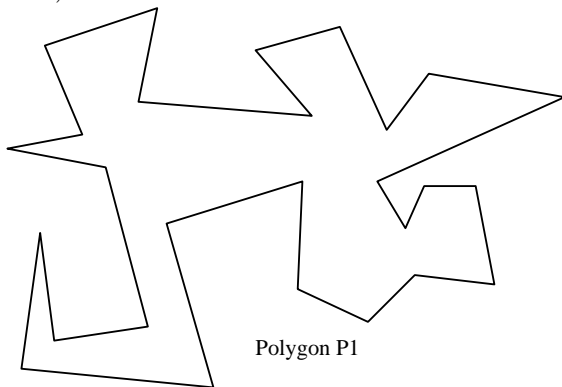
(Optionally, you may download, compile and execute the code from the textbook web site, and submit a printout of the result of the execution. You should also be able, however, to perform the execution “by hand”. Please confirm that your hand calculation matches what the program gives you.)



Also, *draw* the resulting triangulation by plotting the diagonals you obtain.

(5). [25 points] For each of the simple polygons P below, do the following:

- Show a set of diagonals that yield a triangulation of P .
- Apply the method of Fisk's proof to obtain a set of at most $\lfloor n/3 \rfloor$ (vertex) guards. How many guards do you use?
- By inspection, obtain the *vertex* guard number for P ; i.e., find the minimum number of vertex guards necessary to guard the polygon. Justify your answer! In particular, give an argument that fewer guards cannot suffice.
- By inspection, obtain the *point* guard number for P , allowing guards to be placed at *any* point (interior or boundary) of the polygon. Justify your answer! (Give an argument that fewer guards cannot suffice.)



NOTE: In order to assist you in making drawings of the polygons, I place the polygons on the website as files you can download in various formats (xfig, eps, pdf, jpeg).

- Compute the *edge guard number*, $g_e(P)$, which is defined to be the minimum number of *edges* needed to illuminate all of the polygon. (Think of an edge as being a long skinny light bulb; the set of points of P that is seen (its "visibility polygon") is the locus of points in P that see *some* point of the edge.) Make sure to give a lower bound argument showing that your upper bound is tight.