

## COMPUTATIONAL GEOMETRY

### Homework Set # 7

Due at the beginning of class on Thursday, December 6, 2007 *Reminder: Show your reasoning!*

Recommended Reading: BKOS: Chapters 7 and 9 (you may skip 9.5), and Section 11.5; O'Rourke, Chapters 5 and 6.

#### DO ANY 5 OF THE FOLLOWING 7 PROBLEMS.

- (1). Problems 7.5, 7.6, 7.7
- (2). Problem 9.1. (In part (b), I think you can get a lower bound of  $2^{n-2\sqrt{n}+1}$ , better by factor of 2 than what the book gives. Hint: Think of grids.)
- (3). Problem 9.4
- (4). Consider the  $L_\infty$  metric in the plane:  $d_\infty(p, q) = \max\{|p_x - q_x|, |p_y - q_y|\}$ .
  - (a). Draw an example of a “disk” (locus of points at distance at most  $r$ ) in the  $L_\infty$  metric.
  - (b). Describe what the *bisector* (locus of points,  $b(p, q)$ , that are equidistant from  $p$  and from  $q$ ) looks like for the  $L_\infty$  metric in the plane. Draw examples to show different possibilities. (What do you find to be particularly unusual about the “bisectors”, compared with the usual Euclidean case?)
  - (c). Using graph paper (or a computer drawing package, such as xfig or ipe or similar), construct a careful drawing of the Voronoi diagram according to the  $L_\infty$  metric of the following point set:  $p_1 = (-2, 3)$ ,  $p_2 = (-2, -1)$ ,  $p_3 = (3, -1)$ ,  $p_4 = (4, 1)$ ,  $p_5 = (4, 2)$ ,  $p_6 = (4.5, 1.5)$ . Label each cell, each edge, and each vertex of the diagram, classifying each as belonging to “ $V(Q)$ ”, and give the subset of sites  $Q$  (Here, I use the notation on the Voronoi handout I gave in class).  
Also draw the Delaunay diagram.
- (5). (a). Construct the Euclidean Voronoi and Delaunay diagrams for the following point set:  $p_1 = (-2, 3)$ ,  $p_2 = (-2, -1)$ ,  $p_3 = (3, -1)$ ,  $p_4 = (4, 1)$ ,  $p_5 = (4, 2)$ ,  $p_6 = (4.5, 1.5)$ .  
(b). Construct the *furthest neighbor* Euclidean Voronoi and Delaunay diagrams for the following point set:  $p_1 = (-2, 3)$ ,  $p_2 = (-2, -1)$ ,  $p_3 = (3, -1)$ ,  $p_4 = (4, 1)$ ,  $p_5 = (4, 2)$ ,  $p_6 = (4.5, 1.5)$ .  
(c). Construct the Euclidean minimum spanning tree, the relative neighborhood graph, and the Gabriel graph also for the same point set as in parts (a) and (b).
- (6). Problem 9.2 and 9.17. For problem 9.2, give an example with points in general position (no three collinear).
- (7). Given two sets of points,  $A$  and  $B$ , in the plane, design the most efficient algorithm you can to decide if there exists a circle that separates  $A$  and  $B$  (i.e., all of  $A$  lies inside the circle, all points of  $B$  lie outside the circle). How efficient is your algorithm in terms of  $n = |A| + |B|$ , the total number of points?