COMPUTATIONAL GEOMETRY
Homework Set # 8

Due by 4pm, Friday, December 14, 2007. *Reminder: Show your reasoning!*

Recommended Reading: BKOS: Chapters 8 and 10. O'Rourke, Chapter 6.

DO ANY 5 OF THE FOLLOWING 7 PROBLEMS.

(1). [20 points] Problems 8.1, 8.8, BKOS.

(2). [20 points] We have seen how to count the number of vertices, edges, and cells (2-faces) in an arrangement of lines in the plane. Now, let's consider a simple arrangement of planes in 3-space. (By "simple", recall, that we mean that any three planes have a unique point in common, while any four planes have no point in common.) Derive formulas for the number of vertices, edges, facets (2-faces), and cells (3-faces) in a simple arrangement of n planes in 3-space.

(3). [20 points] Problem 8.16, BKOS. While the text asks for expected time/space bounds, you should be able to give deterministic time and space bounds.

(4). [20 points] Problem 8.4, BKOS. Also, find the flaw in the following solution, which a student once proposed:

> *Let $L = \{l_1, ..., l_n\}$ be set of lines in the plane. Straighforward way of computing an axis parallel rectangle that contains all the vertices of the arrangement $\mathcal{A}(L)$ would be finding all the intersection points of all pairs of lines and finding minimal and maximal x and y coordinates. That takes $\mathcal{O}(n^2)$, however.*
>
> *How does the problem translate to dual plane? Imagine an axis-parallel bounding box that is big enough to contain all the vertices of the arrangement. Let's consider finding the lower bound in y-coordinate first. Lines in L in dual plane correspond to n points l_i^*. We can sort them by y-coordinate in $\mathcal{O}(nlogn)$ and find the line C^* given by $y = M$ such that that all the points l_i^* lie below C^*. Since duality preserves order (point is below the line if and only if dual of the line is below the dual of the point), in primal plane point $C(0, -M)$ has to be below all of the lines l_i. Now take an intersection point p of two lines l_i and l_j. Since C is below both of the lines, it is in the specific "south" wedge out of 4 wedges formed by the two intersecting lines. Therefore y-coordinate of C has to be less than y-coordinate of the intersection point p. The same is true for any intersection point so we have found the lower y-coordinate bound. We can find upper y-coordinate bound for the rectangle at the same time (by taking the line D^* given by $y = m$ that lies above all the lines, which gives $-m$ as the upper bound).*
> *We can repeat the symmetrical reasoning and sort points in x-coordinate to find upper and lower limits (that will work in practice but theoretically we imagine rotating the coordinate system by 90 degrees to avoid dealing with vertical lines in dual plane that do not have original in primal plane).*
> *Since the operations of computing the dual of line/point take constant time, the total running time of the algorithm is $\mathcal{O}(nlogn)$.*

(5). [20 points] Let L be a set of n lines in the plane such that no three lines pass through a common point. Assume that exactly k ($k \leq n$) of the lines are all parallel to each other (call this set L_0) and that all of the other $n - k$ lines (in the set $L_1 = L \setminus L_0$) are not parallel to each other or to any line of L_0. Derive formulas for the number of vertices, edges, and faces (cells) in the arrangement, $\mathcal{A}(L)$, of the lines L. Your formulas for v, e, and f will depend on n and k. (Note that your formulas should match what we get for the case $k = 0$, when the set of lines forms a *simple* arrangement.)

(6). [20 points] Problem 10.7(a), BKOS.

(7). [20 points] Problem 10.8, BKOS.