Reeb Graph

“The Reeb graph of \( f \) is obtained by contracting the connected components of the level sets to points....Branching in the Reeb graph occurs only at nodes that correspond to a level set passing through critical points of \( f \).” (Please see K. Cole-McLaughlin, H. Edelsbrunner, J. Harer, V. Natarajan, and V. Pascucci. Loops in Reeb Graphs of 2-Manifolds.)

A Loop is a cycle; the reverse is not necessarily true.

Figure 1: A double torus (with \( f \) equal to the height) and its corresponding Reeb graph. A Loop is a cycle; the reverse is not necessarily true.

Matching

Match the nodes of the Reeb graph according to the following rule:
Each low point is paired with the lowest high point with which it spans a cycle.

Exercise 1. Show that such a matching exists.
Exercise 2. Show that the rule is equivalent to matching each high point to the highest low point with which it spans a cycle.

Exercise 3. Show that such a matching can be found in $O(n \log n)$.

Figure 2: A more interesting example with matching.

Other questions

- Spley trees?
- Reeb graphs and higher dimension manifolds?