Minimum-Link Problems Revisited

Joseph S. B. Mitchell\(^*\) Valentin Polishchuk\(^†\) Mikko Sysikaski\(^†\)

Abstract

In the \(C\)-oriented world—a generalization of the rectilinear one—or-orientations of polygons’ and paths’ edges come from a fixed set of \(C \geq 2\) orientations. We give an \(O(C^2 n \log n)\)-time \(O(Cn)\)-space algorithm to find a minimum-link \(C\)-oriented path in a \(C\)-oriented domain with \(n\) vertices. This improves the algorithms of Adegeest, Overmars and Snoeyink [1] that run in \(O(C^2 n \log n)\) time and space or in \(O(C^2 n \log^2 n)\) time and \(O(C^2 n)\) space. Similarly to [1], the output of our algorithm is a set of \(C\) trapezoidations of the domain, with the trapezoids labeled by link distance to a fixed source point (which enables answering link distance queries to the source in \(O(C \log n)\) time); however, unlike [1], our algorithm uses only basic data structures such as priority queues and binary search trees to label the trapezoids.

We also prove that in the case of unrestricted orientations, finding a minimum-link path is 3SUM-hard; this answers in the negative an open question from the survey of Mitchell [3], and suggests that the only known, nearly-quadratic algorithm of Mitchell, Rote and Woeginger for computing minimum-link paths [2] runs in almost optimal time.

Finally, we show how to find a constant-factor approximation to the minimum-link simple cycle that encloses a simple polygon while staying within a given tolerance away from it.

References


\(*\) Applied Math and Statistics, Stony Brook University. jsbm@ams.stonybrook.edu

\(†\) Helsinki Institute for Information Technology. firstname.lastname@cs.helsinki.fi