AMS 311 (Fall, 2004) Kenny Ye

PROBABILITY THEORY

Practice Exam 2

Write your answers on this examination, using the backs of pages if needed. Separate pages of scratch paper are available if you need it. There may be problems that are solvable by inspection, but if you get the wrong answer and have shown no work, then I will assign NO partial credit.

You do not need to evaluate arithmetic expressions or integrals, if they are fully specified. For example, leave $(2+3+4+\cdots+100)\binom{23}{5}+\sum_{i=3}^{9}\binom{12}{i}(0.8)^i-\frac{10!}{3!4!3!}\times 5!$ in exactly this form, and you can leave $\int_0^{0.5}\int_x^{1-x}x^2e^y dydx$ in this form

This examination is CLOSED BOOK and CLOSED NOTES and NO CALCULATORS.

You are allowed to bring in a letter size (8.5 inch $\times$ 11 inch) cheat sheet.

1. Only one-side of the sheet can be used.
2. Clearly write your name and ID on the sheet.

Work carefully, and GOOD LUCK!!!

Name (PRINT CLEARLY) and ID number:

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PLEASE THINK ABOUT THE STATEMENT BELOW BEFORE YOU SIGN!!

Academic integrity is expected of all students at all times. Cheating will *not* be tolerated in this course.

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I am caught cheating (either giving or receiving unauthorized aid), I will receive a "Q" grade in this course, and a letter will be sent to the Committee on Academic Standing and Appeals (CASA) requesting that an academic dishonesty notation be placed on my transcript. Further action against me may also be taken, as the instructor will do everything in his power to make certain that the maximum applicable penalty is imposed.

Signature:

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Some Possibly Useful Formulas:

Geometric(p): $p(i)=(1-p)^{i-1}p$, $\quad i=1,2,\ldots$. Mean is $1/p$; variance is $(1-p)/p^2$.

Binomial(n,p): $p(i)=\binom{n}{i}p^i(1-p)^{n-i}$, $\quad i=0,1,\ldots,n$. Mean is np; variance is $np(1-p)$.

Poisson(λ): $p(i)=e^{-\lambda}\frac{\lambda^i}{i!}$, $\quad i=0,1,2,\ldots$. Mean and variance equal λ.

NegBin(r,p): $p(n)=\binom{n-1}{r-1}p^r(1-p)^{n-r}$, $\quad n=r,r+1,\ldots$. Mean is r/p; variance is $r(1-p)/p^2$.

Normal(μ,σ^2): $f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$ $\quad -\infty<x<\infty$; $\frac{X-\mu}{\sigma}$ is Normal(0,1).

Exponential(λ): $F(a)=1-e^{-\lambda a}$, $\quad a\ge 0$; $f(x)=\lambda e^{-\lambda x}$ if $x\ge 0$. Mean is $1/\lambda$, variance is $1/\lambda^2$.

$$\int x^n dx=\frac{x^{n+1}}{n+1}\quad (n\neq -1);\qquad \int e^{ax}dx=\frac{e^{ax}}{a}\qquad \int xe^{ax}dx=\frac{e^{ax}}{a^2}(ax-1)$$

1. From an ordinary deck of 52 cards, you select at random 3 of them (without replacement). Let X be the number of spades you get.

Give the probability mass function for X. Be very explicit! (Reminder: you need not evaluate arithmetic expressions.)

1. Consider a random variable X whose cumulative distribution function is given by

$$F(x) = \begin{cases} 0 & \text{if } x < -1 \\ 0.2 & \text{if } -1 \leq x < 0 \\ q & \text{if } 0 \leq x < 1 \\ 0.6 & \text{if } 1 \leq x < 2 \\ 0.7 & \text{if } 2 \leq x < 4 \\ 1 & \text{if } x \geq 4. \end{cases}$$

You are also told that $P(X^2 > 0) = 0.8$.

(a). What is q?

(b). What is $p(0)$? What is $p(-2)$? What is $p(P(X \leq 4.1))$?
(Here, $p(\cdot)$ denotes the probability mass function (pmf) for X)

(c). Sketch a plot of the function $p(x)$ below. (Make sure to label the coordinates on the axes!)

(d). Compute $P(X^2 - 1 \geq 3)$.

(e). What is $P(2X - 3 \geq 2 \mid X \geq 0.1)$?

(f). Compute $E(X)$.

(g). Compute $E(p(X))$.

3. Let X denote the length of time (in hours) it takes a student to complete the AMS 311 midterm exam. Suppose the probability density function of X is given by $f_X(x) = (x - 20)/800$, 20min $< x <$ 60 min.

(a). Compute $var(X)$.

(b). What is the probability that a student take less than 40 minutes to complete the midterm exam?

(c). Suppose now that we estimate that a student who takes time X to complete the midterm will take time $Y = 2X$ to complete the final exam. Determine the cdf, $F_Y(y)$, of Y. Be very explicit! You must show the value of $F_Y(y)$ for *all* values of y; be careful about all cases. (Hint: find $F_Y(y) = P(Y < y) = P(2X < y)$)

(d). Sketch a plot of $F_Y(y)$ here, labelling important values along the axes.

4. On average, 5.2 hurricanes hit a certain region in a year. What is the probability that there will be 3 or fewer hurricanes hitting this year?

5. Let X denote the lifetime of a radio, in years, manufactured by a certain company. Assume that X is exponentially distributed with mean 5 years and that these radios come with a 5-year warranty.

(a). You purchase a radio today. Given that the radio is still working fine two years from today, what is the probability that the radio breaks down before the warranty is up?

(b). What is the (exact) probability that, of 20 such radios that you purchase, at least one of them last more than 15 years? (Hint: use Binomial Distribution.)