3.4. **Independent Events**

Two events $E$ and $F$ are independent, if the happening (or not happening) of one events does not affect the probability of the other. Use conditional probability, this can be written as

$$P(E|F) = P(E), \quad P(F|E) = P(F).$$

Since the definition of conditional probability tells us that $P(E|F) = P(EF)/P(F)$ and $P(F|E) = P(EF)/P(E)$, we have the following definition.

**Definition:** Two events $E$ and $F$ are called independent if

$$P(EF) = P(E)P(F).$$

If two events are not independent, they are called dependent. If $E$ and $F$ are independent, we say that $\{E, F\}$ is an independent set of events.

**Definition:** The three events $E$, $F$, and $G$ are independent if

1. $P(EFG) = P(E)P(F)P(G)$
2. $P(EF) = P(E)P(F)$
3. $P(EG) = P(E)P(G)$
4. $P(FG) = P(F)P(G)$

Events are independent by design:
- Device has been constructed to have independent outcomes (roulette wheels, etc.).
- A sample has been taken following the precise rules.
- Experimental units have been randomly assigned to treatments.

Sometimes need the definition to show two events are independent.

**Example 4a.** A card is selected at random from an ordinary deck of 52 playing cards. If $E$ is the event that the selected card is an ace and $F$ is the event that it is a spade, are $E$ and $F$ independent? Answer: Yes.

**Example 4c.** Suppose that we toss 2 fair dice. Let $E$ denote the event that the sum of the dice is 6. Let $F$ denote the event that the first die equals 4. Let $G$ denote the event the sum of the dice is 7. Are $E$ and $F$ independent? (No). Are $F$ and $G$ independent? (Yes).

**Example 4f.** An infinite sequence of independent trials is to be performed. Each trial results in a success with probability $p$ and a failure with probability $1-p$. What is the probability that

a) At least 1 success occurs in the first $n$ trials;

b) Exactly $k$ successes occur in the first $n$ trials;

c) All trials result in successes?