AMS 311, Spring Semester 2004

3.5 \( P(\cdot \mid F) \) Is a Probability

Proposition 5.1

a) \( 0 \leq P(E \mid F) \leq 1 \).

b) \( P(S \mid F) = 1 \).

c) If \( E_i, i = 1,2,\ldots \) are mutually exclusive events, then \( P(\bigcup_{i=1}^{\infty} E_i \mid F) = \sum_{i=1}^{\infty} P(E_i \mid F) \).

Example 5c. At a party \( n \) men take off their hats. The hats are then mixed up, and each man randomly selects one. We say that a match occurs if a man selects his own hat. What is the probability of

a) No matches;

b) Exactly \( k \) matches?

A straightforward solution is given in example 5m in Chapter 2.

a) Let \( E_i \) be the event that the \( i \)th man has his hat. Compute \( 1 - P(\bigcup_{i=1}^{n} E_i) \) using inclusive-exclusive formula.

b) Use results of a).

Example 4j. Gambler’s Ruin Problem

Two gamblers, \( A \) and \( B \), bet on the outcomes of successive flips of a coin. On each flip, if the coin comes up heads, \( A \) collects 1 unit from \( B \), whereas if it comes up tails, \( A \) pays 1 unit to \( B \). They continue to do this until one of them runs out of money. If it is assumed that the successive flips of the coin are independent and each flip results in a head with probability \( p \), what is the probability that \( A \) ends up with all of the money if he starts with \( i \) units and \( B \) starts with \( N-i \) units.

Let \( P_i \) be the probability that \( A \) ends up with all the money. Then establish the following relation \( P_i = pP_{i+1} + (1-p)P_{i-1} \). Use \( P_0 = 0 \) and \( P_N = 1 \), we can find probability of \( P_i \).