Chapter Four: Random Variables

4.1. Random Variables

A random variable is a real valued function defined on the sample space that satisfies certain conditions. (Rigorous definition requires measure theory arguments and will not be given here.)

In the case of discrete random variable, a sample space is partitioned to a number of events. Each event corresponds to a real value that describes this event.

Definition: If X is a random variable, then the function F defined on $(-\infty, \infty)$ by $F(t) = P(X \le t)$ is called the **cumulative distribution function** of X.

Some authors use the term distribution function rather than cumulative distribution function (cdf).

4.2. Discrete Random Variables

For a discrete random variable X, we define the probability mass function (pmf) $p(a)$ of X by $p(a) = P\{X = a\}$.

Example 2a. The probability mass function of a random variable X is given by $p(i) = \frac{c\lambda^i}{i!}$, $i = 0, 1, 2, \ldots$, where $\lambda > 0$. Find $P\{X = 0\}$ and $P\{X > 2\}$.

4.3. Expected Value

Definition The expected value of a discrete random variable X with the probability function $p(x)$ and set of possible values A (that is, those values x with $p(x)>0$) is defined by

$$E(X) = \sum_{x \in A} xp(x).$$

We say that $E(X)$ exists if this sum converges absolutely.

If X is a constant random variable, that is, if $P(X = c) = 1$ for a constant c, then $E(X) = c$.

4.4. Expectation of a Function of a Random Variable

Example 4a. Let X denote a random variable that taken on any of the values –1, 0, 1 with respective probabilities $p(-1) = P\{X = -1\} = 0.2$, $p(0) = P\{X = 0\} = 0.5$, $p(1) = P\{X = 1\} = 0.3$. Compute $E(X^2)$.

Proposition 4.1. Let *X* be a discrete random variable with set of possible values *A* and probability function *p(x)*, and let *g* be a real-valued function. Then *g(X)* is a random variable with

$$E[g(X)] = \sum_{x \in A} g(x)p(x).$$

Corollary 4.1. If *a* and *b* are constants, then $E(aX + b) = aE(X) + b$.

Corollary Let *X* be a discrete random variable; $g_1, g_2, \ldots, g_n$ be real-valued functions, and let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be real numbers. Then

$$E[\alpha_1 g_1(X) + \alpha_2 g_2(X) + \cdots + \alpha_n g_n(X)] = \alpha_1 E[g_1(X)] + \alpha_2 E[g_2(X)] + \cdots + \alpha_n E[g_n(X)].$$

End Handout