AMS 311 (Fall, 2004) Jasmin Divers

PROBABILITY THEORY

Answer to exam 2

Problem 1

a) *Finding q*

From the CDF of X we have:

$P(X=-2)=0.2, P(X=0)=q-0.2, P(X=1)=1-q.$

Using the fact that E(X)=0, we can solve for q. By definition,

$E(X)=\sum_{allx} xP(X=x)$ This is equivalent to

$-2*0.2+0*(q-0.2)+1*(1-q)=0 \Rightarrow -0.4+1-q=0.$

Therefore, q=0.6.

b)Bar graphs with X on the X-axis and P(X=x) on the Y-axis.

c)$P(X^2 \geq 2|X \geq 0)=1-P(X^2 \leq 2|X \geq 0)$

$=1-P(-\sqrt{2} \leq X \leq \sqrt{2}|X \geq 0)$

$=1-\frac{P(0 \leq X \leq \sqrt{2})}{P(X \geq 0)}$

$=1-\frac{P(X=0)+P(X=1)}{P(X=0)+P(X=1)}=1-1=0$

$E(\frac{1}{X+1})=\sum_{allx} \frac{1}{X+1}P(X=x)$

$E(\frac{1}{X+1})=\frac{1}{-2+1}*0.2+\frac{1}{0+1}*0.4+\frac{1}{1+1}*0.4$

$=-0.2+0.4+0.2=0.4$

Problem 2

a) *Finding c*

c is such $\int_{100}^{\infty} \frac{c}{x^2}dx=1 \Rightarrow \frac{c}{100}=1 \Rightarrow c=100.$

b) CDF of Y=2X+10

$F_y(Y)=P(Y<y)=P(2X+10<y)=P(2X<y-10)=P(X<\frac{y-10}{2})$

$=F_x(\frac{y-10}{2})=\int_{100}^{\frac{y-10}{2}} \frac{100}{x^2}dx=100(\frac{1}{100}-\frac{2}{y-10})=\frac{y-210}{y-10}=1-\frac{200}{y-10}.$

Therefore, the CDF of Y is written as follows:

$$F_y(y)=\begin{cases} 0 & y<210 \\ 1-\frac{200}{y-10} & 210 \leq Y < \infty \end{cases}$$

c) Taking derivative of $F_y(y)$ with respect to y, we have:

$$f_y(y) = \begin{cases} \frac{200}{(y-10)^2} & 210 \leq y < \infty \\ 0 & Otherwise \end{cases}$$

Problem 3

Let X represent the number of car accidents that happen today. X follows a Poisson ($\frac{3}{7}$).
$P(X \geq 2) = 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1)) = 1 - e^{\frac{-3}{7}} - 3e^{\frac{-3}{7}}$
$= 1 - 4e^{\frac{-3}{7}}$.

Problem 4
Let X represent the time (in hours) that it takes to repair a machine X follows an Exp(1)
a) $P(X > 2) = \int_2^{\infty} e^{-x} dx = e^{-2}$
b) $P(X < 5|X > 4) = 1 - P(X > 5|X > 4) = 1 - P(X > 1) = 1 - \int_1^{\infty} e^{-x} dx = 1 - e^{-1}$

Bonus
Distribution of $-log(X)$

$F_y(Y) = P(Y < y) = P(-log(X) < y) = P(log(X) > -y) = P(X > e^{-y})$
$= 1 - F_x(e^{-y}) = 1 - \int_0^{e^{-y}} dx = 1 - e^{-y}$.
Which is the CDF of an exponential distribution with $\lambda = 1$.