

PROBABILITY THEORY

Homework Set # 3

Due at the Midterm Exam, Feb 25, 2003. *Reminder: Show your reasoning!*

Read: Ross, Chapter 3, sections 3.1-3.4.

- (1). (16 points) Consider an experiment in which two fair dice are rolled. What is the conditional probability that their sum is 6, given that the dice land on same numbers? What is the conditional probability that the dice land on same numbers given that their sum is 6. (Hint: write the sample space of all possible outcomes of two dice, then count).
- (2). (16 points) Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determine the probability that each hand has an ace. Let E_i be that the i th hand has exactly one ace. Determine $P(E_1E_2E_3E_4)$ by using the multiplication rule. (Hint: this is the same problem as I did in class but here gives a different approach.)
- (3). (16 points)
 - (a). A gambler has in his pocket two fair coins and one two-headed coin. He selects one of the coins at random; when he flips it, it shows heads. What is the probability that it is a fair coin? (hint: Use Bayes' formula)
 - (b). Suppose that he flips the same coin a second time and again it shows heads. What is now the probability that it is a fair coin? (hint: for the fair coin, assumes the flips are independent, the probability of getting two heads in two flips equals $0.5 \times 0.5 = 0.25$. for the two-headed coin, the probability of getting two heads in two flips is 1.)
 - (c). Suppose that he flips the same coin a third time and it shows tails. What is now the probability that it is a fair coin?
- (4). (20 points) Suppose that an insurance company classifies people into one of three classes: good risks, average risks and bad risks. The record indicate that the probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are, respectively, 0.05, 0.15 and 0.25. If 30 percent of the population are good risks, 50 percent are average risks, and 20 percent are bad risks, what proportion of people have accidents in a fixed year? If a policyholder Bob had no accidents in 2003, what is the probability that he or she is a good (average) risk?
- (5). (16 points) A true-false question is to be posed to a husband and wife team on a quiz show. Both the husband and the wife will, independently, give the correct answer with probability $p = 0.6$. The couple decided to use the following strategy to give their answers. Both of them consider the question independently and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give.
 - (a) What is the conditional probability that the couple gives the correct answer given that they *agree*?
 - (b) What is the conditional probability that the couple gives the correct answer given that they *disagree*?
 - (c) What is the probability that their answer is correct.
- (6). (16 points) Suppose we roll a red and a green die. Let $A =$ "The red die shows a 2 or a 4," $B =$ "The sum of the two dice is at most 7." Are events A and B independent?