

PROBABILITY THEORY

Correct answer for problem 3

Sorry guys! For some reason I read two coins: one fair and a two-headed one.

Also, a typo in problem 4 is corrected. The correct answer to 4b is given below.

Problem 3

The probability that the gambler choose the fair coin is $\frac{2}{3}$. Also note that if the fair coin is flip, the probability of a head is $\frac{1}{2}$. However, if the two-headed coin is flip, the probability of getting a head is always equal to 1. Let "fair" be the event that the die is fair and let "head" be the event that the flipped coin landed on head.

$$a) P(\text{fair}/\text{head}) = \frac{P(\text{fair}) \times P(\text{head}/\text{fair})}{P(\text{fair}) \times P(\text{head}/\text{fair}) + P(\text{not fair}) \times P(\text{head}/\text{not fair})}$$

$$P(\text{fair}/\text{head}) = \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{3} \times 1} = \frac{1}{2}$$

$$b) P(\text{fair}/\text{head, head}) = \frac{P(\text{fair}) \times P(\text{head, head}/\text{fair})}{P(\text{fair}) \times P(\text{head, head}/\text{fair}) + P(\text{not fair}) \times P(\text{head, head}/\text{not fair})}$$

$$P(\text{fair}/\text{head}) = \frac{\frac{1}{4} \times \frac{2}{3}}{\frac{1}{4} \times \frac{2}{3} + \frac{1}{3} \times 1} = \frac{1}{3}$$

c) Tail can only come from the fair coin, $P(\text{fair}/\text{outcome1, outcome2, tail})=1$.

Problem 4

Let A be the event that a person will be involved in a car accident. Let GR, AR, LR represent respectively the good, average and low risk groups. $P(A/GR)=0.05$, $P(A/AR)=0.15$ and $P(A/LR)=0.25$ also, $P(GR)=0.3$, $P(AR)=0.5$ and $P(LR)=0.2$. First, we need to find $P(A)$.

$$P(A) = P(A/Gr)P(GR) + P(A/AR)P(AR) + P(A/LR)P(LR) = 0.05 \times 0.3 + 0.15 \times 0.5 + 0.25 \times 0.2 = 0.14.$$

$$P(GR/\bar{A}) = \frac{P(GR) \times P(\bar{A}/GR)}{P(\bar{A})} = \frac{P(GR) \times (1 - P(A/GR))}{1 - P(A)} = \frac{(0.3) \times (1 - 0.05)}{1 - 0.14} = 0.3313$$