PROBABILITY THEORY
Homework Set # 6

Due at the beginning of class on Thursday, October 30, 2003. Reminder: Show your reasoning!
Read: Ross, Chapter 5, Sections 5.1–5.5, 5.7. (You may skip section 5.5.1 on hazard rate functions.)

(1). (20 points) The density function of $X$ is given by

$$f(x) = \begin{cases} a + bx^2 & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Suppose also that you know that $E(X) = 1.5$.
(a). Find $a$ and $b$; (Hint: You need two equations to solve for $a$ and $b$. One of them is from $E(X) = 1.5$.)
(b). Determine the cdf, $F(x)$, explicitly.
(c). Find $P(1 < X < 2)$.

(2). (20 points) The lifetime (in hours) of a lightbulb is an exponentially distributed random variable with parameter $\lambda = 0.10$ (units of hours$^{-1}$).
(a). What is the probability that the light bulb is still burning one day after it is installed?
(b). Assume that the bulb was installed at noon today and assume that at 10:00pm today you notice that the bulb is still working.
(i). What is the chance that the bulb will burn out at some time between 12:00 am and 6:00am tomorrow morning? (hint: exponential distribution is memoryless.)
(ii). What is the expected time when the bulb burns out? (again, given that it was still working at 10:00pm today)

(3). (20 points) Suppose that $X$ has density $f_X(x)$ given below.

$$f_X(x) = \begin{cases} x/2 & \text{if } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Let $Y = 3X + 1$.
(a). Compute the cdf and the density of $Y$. (Remember to be very explicit about all cases!) Plot them.
(b). Compute the variance of $Y$ and of $Y^2$.

(4). (20 points) Suppose that $X$ is a normal random variable with parameters $\mu = 3$, $\sigma^2 = 25$. Using the table of the normal distribution (page 203), compute: (a). $P\{X > 6\}$, (b). $P\{4 < X < 7\}$, (c). $P\{X < 8\}$, (d). $P\{2X - 7 < 10\}$.

(5). (20 points) Consider a country in which every couple will have at least one child after getting married, and the time between the wedding and the birth of the first child is an exponential random variable. You know that 10% of those new weds will not have their first child within the next three years. For a couple who have not had a child three years after their wedding, they will not have their child within the next 2 years? (Hint: Let $X$ be the time between the wedding and birth of the first child. It follows an exponential distribution. First find the parameter of the exponential distribution. Then find $P(X > 5 | X > 3)$.)