AMS 311 (Spring, 2004)  
Kenny Ye

PROBABILITY THEORY: Homework Set # 8

Due at the beginning of class on Wednesday, May 5, 2003. Reminder: Show your reasoning!

Read: Ross, Chapter 6, 6.7, Chapter 7, Section 7.2, 7.3, and Section 7.6

SPECIFICS OF READING ASSIGNMENT:
Examples to read carefully:
Chapter 6: 7b, 7d
Chapter 7: 2d, 2f, 2h, 3a, 3b, 3c, 6a, 6b, 6d, 6e, 6f, 6g, 6h

(1). DO ONE OF THE FOLLOWING TWO PROBLEMS. You are responsible to be able to do both of them. One extra credit is given if both are submitted and done correctly.
(a) Ross, Page 295, 54.
(b) Ross, Page 295, 58.

(2). (20 points) First a point $Y$ is selected according to the density

$$f_Y(y) = \begin{cases} 
  y & \text{if } 0 < y < \sqrt{2} \\
  0 & \text{otherwise}
\end{cases}$$

Then another point $X$ is selected (uniformly) at random from the interval $(0, Y + 1)$.

(a). Find the joint probability density function for $X$ and $Y$. (Hint: First find $f_{X|Y}(x|Y = y)$, then use $f_{X,Y}(x,y) = f_{X|Y}(x|Y = y) f_Y(y)$ to obtain $f_{X,Y}(x,y)$. Pay great attention of the domain of $f_{X,Y}(x,y)$)
(b). Find the (marginal) probability density function of $X$.

(3). If $X$ and $Y$ are independent uniform $(0,1)$ random variables, Find $E[(X - Y)^2]$.

(4). DO ONE OF THE FOLLOWING TWO PROBLEMS. You are responsible to be able to do both of them. One extra credit is given if both are submitted and done correctly.
(a) Ross, page 384, 37 (Hint, first find the joint distribution of $X$ and $Y$)
(b) Ross, page 384, 38 (Hint, watch out the domain where $f(x,y) \neq 0$)

(5). DO ANY ONE OF THE FOLLOWING 2 PROBLEMS; You are responsible to be able to do all of them. One extra credits is given if both are submitted and done correctly.
(a) Suppose $X$ takes on values -1, 3, and 7, each with probability 1/3. (i) Compute the moment generating function, $M_X(t)$ of $X$. (ii). Use the moment generating function to compute $E(X)$ and $E(X^2)$, and verify your answers by computing the values directly. (b) (20 points) The moment generating function of $X$ is given by $M_X(t) = e^{2e^{t-2}}$ and that of $Y$ is given by $M_Y(t) = (\frac{3}{2}e^t + \frac{1}{2})$. If $X$ and $Y$ are independent, what are (i). $P(XY = 0)$; (Hint: Use Table 7.1 to find out the distribution of $X$ and $Y$, then use basic rules of probability.) (ii). $E(XY)$. (Hint: $X$ and $Y$ are independent, and $Cov(X,Y) = 0$.)