AMS 311 (Spring, 2004) Kenny Ye

PROBABILITY THEORY

Practice Exam 3

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 6 pages (including this cover sheet) of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed. Separate pages of scratch paper are available if you need it. There may be problems that are solvable by inspection, but if you get the wrong answer and have shown no work, then I will assign NO partial credit.

You do not need to evaluate arithmetic expressions or integrals, if they are fully specified. For example, leave $(2+3+4+\cdots+100)\binom{23}{5}+\sum_{i=3}^{9}\binom{12}{i}(0.8)^i-\frac{10!}{3!4!3!}\times 5!$ in exactly this form, and you can leave $\int_0^{0.5}\int_x^{1-x} x^2e^y dydx$ in this form

This examination is CLOSED BOOK, CLOSED NOTES, and NO CALCULATORS. You can bring a cheat sheet of the letter size (8×11.5).

CELL PHONES must be OFF (NOT on vibrate alert), and put out of sight If you have no watch and need updates of the time, please ask.

Last Name (PRINT CLEARLY): ____________________

First Name (PRINT CLEARLY): ____________________

ID Number: ____________________

PLEASE THINK ABOUT THE STATEMENT BELOW BEFORE YOU SIGN!!

Academic integrity is expected of all students at all times. Cheating will *not* be tolerated in this course. Students observing any act of academic dishonesty are requested to notify the instructor (discretely and anonymously at an appropriate time, in order to protect privacy).

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I cheat I will be subjected to the maximum possible penalty permitted under University guidelines, including dismissal from the University. I understand that the instructor will do everything in his power to make certain that the maximum applicable penalty is imposed.

Signature:

Some Possibly Useful Formulas:

Markov inequality: $P\{X\geq a\}\leq \frac{\mu}{a}$. Here, $a>0$, $P\{X\geq 0\}=1$.

Chebyshev inequalities: $P\{|X-\mu|\geq k\}\leq \frac{\sigma^2+}{k^2}$.

$$\int x^n dx=\frac{x^{n+1}}{n+1}\quad (n\neq -1);\qquad \int e^{ax}dx=\frac{e^{ax}}{a}\qquad \int xe^{ax}dx=\frac{e^{ax}}{a^2}(ax-1)$$

1. Let X and Y be continuous random variables with joint probability density function given by

$$f(x,y) = \begin{cases} \frac{3}{4}(2-x-y) & \text{if } x+y<2,\ 0<x<2,\ 0<y<2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the following quantities; you may leave any integrals unevaluated, BUT you must write them completely, with proper limits of integration, "dx", "dy", etc.

(a). (6 points) The marginal density, $f_Y(y)$, of Y. (Be explicit about all cases.)

(b). (7 points) Compute the conditional distribution of X at $Y = 0.5$.

(c). (7 points) Compute $E(X \mid Y = 0.5)$

2. The moment generating function of the random variable X is given by $M_X(t) = \frac{e^{3t}}{1-25t^2}, |t| < 0.2$.

(a) (5 points) Compute $E(X)$.

(b) (15 points) Compute $Var(X)$.

3. On a single tank of gasoline, you expect to be able to drive 240 miles before running out of gas.

(a). (10 points) Let p be the probability that you will **NOT** be able to make a 300 mile journey on a single tank of gas. What can you say about p? (Hint, use Markov's inequality.)

(b). (10 points) Assume now that you also know that the standard deviation of the distance you can travel on a single tank of gas is 30 miles. What can you say now about p? (Hint, use Chebyshev's inequality.)

4. (20 points) The random variables X and Y are continuous with joint probability density function when

$$f(x,y) = \begin{cases} \frac{1}{6}xe^{-x}y^2e^{-y} & \text{if } x > 0,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

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Compute the joint density of $U = X + Y$ and $V = X/Y$. Indicate the region over which $f_{U,V}(u, v)$

5. The random variables X and Y are jointly distributed such that $E(X|Y) = 5Y$ and $Var(X|Y) = 4$ and Y has the marginal probability density function $f_Y(y) = e^{-y}, y > 0$ and zero otherwise.

(a) (10 points) Compute $E(X)$. (Hint: $E(X) = E[E(X|Y)]$.)

(b) (10 points) Compute $Var(X)$. (Hint: $Var(X) = Var[E(X|Y)] + E[Var(X|Y)]$)