Answer to the practice Exam.

Problem 1

a) Probability that you get exactly 2 Milkyway bars and 3 bags of Skittles.

$$\frac{\binom{7}{2}\binom{12}{3}}{\binom{22}{5}} = \frac{4620}{26334} = 0.1754$$

b) Probability to get the same number of Milkyway bars and bags of Skittles

$$\frac{\binom{3}{3}\binom{7}{1}\binom{12}{1} + \binom{3}{1}\binom{7}{2}\binom{12}{2}}{\binom{22}{5}}$$

$$= \frac{4242}{26334} = 0.1611$$

Problem 2

a) Let D be the event that a person has the disease. $P(D) = 0.05$.

Let $+$ be the event that the test is positive.

$P(+|D) = 0.99$ and $P(+|\bar{D}) = 0.99$.

b) You want to compute $P(D|+)$

c) See Part (a) for values of $P(D)$, $P(+|D)$ and $P(+|\bar{D})$

d) Probability that a randomly chosen person is tested positive.

$$P(+) = P(+|D)\,P(D) + P(+|\bar{D})\,P(\bar{D})$$

$$P(+) = 0.99 * 0.05 + 0.99 * (1-0.05)$$

$$P(+) = 0.99$$

e) Probability of having the disease if the test is positive

$$P(D|+) = \frac{P(+|D)\,P(D)}{P(+)} = \frac{0.99 * 0.05}{0.99} = 0.05$$

Problem 4

a) Let O be the event that flashlights gives over 100 hours.

Let F_i, $i = 1, 2, 3$ be the event that flashlight is of type i. We have: $P(O|F_1) = 0.7$, $P(O|F_2) = 0.4$, $P(O|F_3) = 0.3$.

b) We want to compute $P(O)$.

c) $$P(O) = \sum_{i=1}^{3} P(O|F_i)\, P(F_i)$$

$P(F_1) = 0.2$, $P(F_2) = 0.3$ and $P(F_3) = 0.5$.

$$P(O) = 0.7 * 0.2 + 0.4 * 0.3 + 0.3 * 0.5$$

$$P(O) = 0.14 + 0.12 + 0.15$$

$$P(O) = 0.41$$

Problem 3

Let $A =$ "The red die shows a 2 or a 4"

$B =$ "The sum of the two dice is at most 7"

a) $P(A) = P(2) + P(4) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$B = \{(1,1), (1,2), (1,3); (1,4); (1,5); \underline{(2,1)}; \underline{(2,2)}; (1,6); \underline{(2,3)}$
$\underline{(2,4)}; \underline{(2,5)}; (3,1); (3,2); (3,3); (3,4); \underline{(4,1)}; \underline{(4,2)}$
$\underline{(4,3)}; (5,1); (5,2); (6,1)\}.$

b) $P(B) = \frac{21}{36}$

c) The two events mutually exclusive since $P(A \cap B) \neq 0$

It is easy to see that the set $A \cap B$ is not empty.

Let the first number represent the value on the red die

$A \cap B = \{(2,5); (4,3); (2,1); (2,2); (2,3); (2,4);$
$(4,1); (4,2)\}$

$P(A \cap B) = \frac{8}{36}$

d) A and B are not independent since $P(A \cap B) \neq P(A)\,P(B)$

$\frac{8}{36} \neq \frac{1}{3} * \frac{21}{36}$

e) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{21}{36} - \frac{8}{36}$

$= \frac{12}{36} + \frac{21}{36} - \frac{8}{36} = \frac{25}{36}$