Answer to the practice Exam.

Problem 1

a) Probability that you get exactly 2 Milkyway bars and 3 bags of Skittles.

\[
\frac{\binom{7}{2} \binom{12}{3}}{\binom{22}{5}} = \frac{4620}{26334} = 0.1754
\]

b) Probability to get the same number of Milkyway bars and bags of Skittles

\[
\binom{3}{3} \binom{7}{1} \binom{12}{1} + \binom{3}{1} \binom{7}{2} \binom{12}{2}
\]

\[
\frac{\binom{22}{5}}{26334} = \frac{4242}{26334} = 0.1611
\]
Problem 2

a) Let $D$ be the event that a person has the disease. $P(D) = 0.05$.
Let $+$ be the event that the test is positive. $P(+|D) = 0.99$ and $P(+|\bar{D}) = 0.99$.

b) You want to compute $P(D|+)$. 

c) See Part (a) for values of $P(D)$, $P(+|D)$ and $P(+|\bar{D})$.

d) Probability that a randomly chosen person is tested positive:

$$P(+) = P(+|D)P(D) + P(+|\bar{D})P(\bar{D})$$

$$P(+) = 0.99 \times 0.05 + 0.99 \times (1 - 0.05)$$

$$P(+) = 0.99$$

e) Probability of having the disease if the test is positive:

$$P(D|+) = \frac{P(+|D)P(D)}{P(+)} = \frac{0.99 \times 0.05}{0.99} = 0.05$$
Problem 4

a) Let $O$ be the event that flashlights give over 100 hours.

Let $F_i$, $i = 1, 2, 3$ be the event that flashlight is of type $i$. We have: $P(O|F_1) = 0.7$, $P(O|F_2) = 0.4$, $P(O|F_3) = 0.3$.

b) We want to compute $P(O)$.

c) $P(O) = \sum_{i=1}^{3} P(O|F_i) P(F_i)$

$P(F_1) = 0.2$, $P(F_2) = 0.3$ and $P(F_3) = 0.5$.

$P(O) = 0.7 \times 0.2 + 0.4 \times 0.3 + 0.3 \times 0.5$.

$P(O) = 0.14 + 0.12 + 0.15$

$P(O) = 0.41$
Problem 3

Let $A = "\text{The red die shows a 2 or a 4}"$

$B = "\text{The sum of the two dice is at most 7}"$

a) $P(A) = P(2) + P(4) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$B = \{ (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (6, 1) \}$

b) $P(B) = \frac{21}{36}$

c) The two events mutually exclusive since $P(A \cap B) = 0$

It is easy to see that the set $A \cap B$ is not empty.

Let the first number represent the value on the red die.

$A \cap B = \{ (2, 5), (4, 3), (2, 1), (2, 2), (2, 3), (2, 4), (4, 2) \}$

$P(A \cap B) = \frac{8}{36}$

d) $A$ and $B$ are not independent since $P(A \cap B) \neq P(A) \cdot P(B)$

$\frac{8}{36} \neq \frac{1}{3} \times \frac{21}{36}$

e) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{21}{36} - \frac{8}{36}$

$= \frac{12}{36} + \frac{21}{36} - \frac{8}{36} = \frac{25}{36}$