AMS 527, S'08, HW 3

1. Derive a formula for $f^{(3)}(a)$ using a Newton interpolating polynomial of third order. Let the interpolating points be $x_0 = a$, $x_1 = a - h$, $x_2 = a + h$, $x_3 = a + 2h$. Derive the error term for the formula.

2. Consider the rule

$$D(f) = \frac{-f(x-h) - 3f(x) + 5f(x+h) - f(x+2h)}{4h}.$$

a) What derivative of $f(x)$ does this rule approximate?

b) What is the error in this approximation?

3. Consider the function $f(x)$ defined as

$$f(x) = \begin{cases} x & 0 \le x \le 1/2 \\ 1 - x & 1/2 \le x \le 1 \end{cases}.$$

Calculate the results of applying the following rules to find $\int_0^1 f(x)dx$:
(a) The trapezoid rule over the interval $[0, 1]$;
(b) The trapezoid rule over the interval $[0, 1/2]$ and then over the interval $[1/2, 1]$;
(c) Simpson's rule over the interval $[0, 1]$;
(d) The corrected trapezoid rule over the interval $[0, 1]$.
Note, in each case DO NOT use a composite rule over the stated interval, i.e. do not subdivide the stated intervals into a smaller partition. For EACH case explain the result.

4. Use the trapezoid rule on a single interval to estimate the value of the integral

$$I = \int_0^1 xe^{-x^2} dx.$$

Obtain a bound on the (single interval) error term for the trapezoid rule and compare with the actual error.

5. Construct a rule of the form

$$I(f) = \int_{-1}^{1} f(x)dx \approx A_0 f(-1/2) + A_1 f(0) + A_2 f(1/2)$$

which is exact for all polynomials of degree ≤ 2.

6. Lobatto's rule for integrating $I(f) = \int_{-1}^{1} f(x)dx$ has the form

$$I(f) \approx A_0 f(-1) + A_1 f(x_1) + \ldots + A_{k-1} f(x_{k-1}) + A_k f(1).$$

Derive the Lobatto rule for $k = 2$ that is exact for all polynomials of degree ≤ 3.

7. Work out the explicit form of the Romberg integration rules $A_{1,1}$, $A_{2,2}$, and $A_{3,3}$. $A_{1,1}$ is a familiar rule - what is it? ($A_{2,2}$ is called Milne's rule.)