AMS 527, S'08, HW 4

1. Derive the normal equations required to solve for a_1 and a_2 in order to fit the functional form

$$F(x;c) = a_1 e^{a_2 x}$$

to tabulated data $(x_i, y_i) \quad i = 0, \ldots m$.

2. By direct computation, verify that the Legendre polynomial $L_3(x)$ is orthogonal to any polynomial of degree 2.

3. Generate the first five Laguerre polynomials.

4. Using the three term recurrence relation, prove that the Legendre polynomial of degree k satisfies,

$$\int_{-1}^{1} [P_k(x)]^2 dx = \frac{2}{2k+1}.$$

5. Let $P_0(x), P_1(x), \ldots$ be a sequence of orthogonal polynomials. Let $x_0, \ldots, x_k$ be the $k+1$ distinct roots of $P_{k+1}(x)$. Prove that the Lagrange polynomials $l_i(x), i = 0, \ldots, k$ for these points are orthogonal to each other.
Hint: Show that, for $i \neq j$, $l_i(x) l_j(x) = P_{k+1}(x) g(x)$, where $g(x)$ is some polynomial of degree $\leq k$.

6. Calculate the polynomial of degree ≤ 2 which is the closest approximation to $\sin(\pi x)$ wrt the weight function $w(x) = 1$.

7. Calculate

$$\int_0^1 \frac{sin(\pi x)}{[x(1-x)]^{3/2}} dx$$

correct to four significant digits.