AMS 527, S'08, HW 5
Due Wed May 14, 10:30 AM

1. Solve

$$u_{n+2} - 4u_{n+1} + 3u_n = 2^n n^2, \qquad u_0 = 0, \qquad u_1 = 1.$$

2. Consider the family of linear multistep methods

$$u_{n+1} = \alpha u_n + (1-\alpha)u_{n-1} + 2hf_n + \frac{h\alpha}{2}\left[f_{n-1} - 3f_n\right].$$

For this family of methods, analyze, as a function of $\alpha \in R$,

a) the order of consistency,

b) the zero-stability.

c) For that value of α for which the method has the greatest order of consistency, analyze the absolute-stability.

3. The Nystrom-Bashforth NB(p,r) methods are obtained by generalizing Adams-Bashforth AB(p) methods by integrating from t_{n-r} to t_{n+1} for $r \geq 1$. (Thus we can identify AB(p) as NB(p,0)).

a) Using the Lagrange form of the interpolating polynomial, show that the NB(p,r) methods have the form

$$u_{n+1} = u_{n-r} + h\sum_{j=0}^{p} b_j f_{n-j}.$$

In particular show that the coefficients b_j do not depend on h.

b) Using either the Lagrange or Newton form of the interpolating polynomial, explicitly work out the form of the NB(2,2) method.

c) How does the order of consistency of NB(2,2) compare with that of AB(2)?