AMS-681 Special Topics in Computational Finance

Assignment 1

(1). Using Wekipedia to find the following topics:

- Option Style
- Binomial Distribution
- Central Limit Theorem
- Markov Process
- Random Walk
- Brownian Motion
- Ito's Lemma
- Black-Scholes Equation

Write a reading digest about these topics.

(2). The Black-Schole equation for European call is

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

with boundary and initial conditions

$$C(0,t) = 0, \qquad C(s,t) \sim S, \quad \text{as} \quad S \to \infty$$

$$C(S,T) = \max(S - E, 0).$$

Transform the problem to the following:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2}, \qquad -\infty < x < \infty,\ \tau > 0,$$

by using the following substitutions

$$x = \log\frac{S}{E}, \quad \tau = \frac{1}{2}\sigma^2(T - t),$$

$$C(x,\tau) = Ee^{-\frac{1}{2}(k-1)x - \frac{1}{4}(k+1)^2\tau}u(x,\tau),$$

where $k = 2r/\sigma^2$. Perform the transformation step by step. What will be the new initial and boundary conditions?

(3). Using the analytical solution for European calls

$$C(S,t) = SN(d_1) - Ee^{-r(T-t)}N(d_2),$$

where

$$d_1 = \frac{\log(S/E) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}},$$

$$d_2 = \frac{\log(S/E) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}},$$

plot the price of the call as a fuction of S with: $(r, \sigma, T) = (0.05, 0.1, 4)$ at $t = 0, 1, 2, 3$.

(4). Using the put-call parity:

$$C - P = S - Ee^{-r(T-t)}$$

plot the price of put for the same stock with the same parameters in (3) at $t = 0, 1, 2, 3$.