AMS-691

Assignment 2

(1). Use the three schemes, explicit, implicit and Crank-Nicolson schemes to solve the parabolic equation $u_t = u_{xx}$. Solve the problem in $[-10, 10]$ to $t = 24$, use grid size 200, 400, 800, 1600 and appropriate CFL condition for stability, if needed. Tabulate the numerical error and convergence order for each scheme. Use the following initial conditions

(a).

$$u(x,0) = \frac{1}{\sqrt{4\pi}} e^{-x^2/4}$$

The exact solution should be:

$$u(x,t) = \frac{1}{\sqrt{4\pi(t+1)}} e^{\frac{-x^2}{4(t+1)}},$$

Use this for both error calculation and boundary conditions.

(b).

$$u(x,0) = \begin{cases} 1 & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases},$$

The exact solution should be:

$$u(x,t) = 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{x}{\sqrt{4t}}} e^{-y^2} dy$$

Use this for both error calculation and boundary conditions.

(2). The European call option is governed by the Black-Schole equation

$$\frac{\partial C}{\partial \tau} - rS\frac{\partial C}{\partial S} = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC$$

where $\tau = T - t$ is the time (in years) to the expiry date, S is the value of the commodity asset related to the call option, $C(\tau, S)$ is the value of the call at time

τ and asset value S, σ is the volatility and r is the interest rate. The expiry value condition is

$$C(0,S) = \begin{cases} S - 2.0 & \text{if } S > 2 \\ 0 & \text{otherwise} \end{cases}$$

and the boundary condition $C(0,\tau) = 0$ and $C(S,\tau) \approx S$ as $S \to \infty$.

(a). Use the explicit, implicit and Crank-Nicolson schemes (for both hyperbolic and parabolic parts) and modify the sample program to solve this equation.

(b). Give the volatility as $\sigma = 0.2$ and interest rate $r = 0.08$, using your scheme to solve the problem in the range of $0 \le S \le 10$ with 200, 400, 800, 1600 grid points. Choose your Δt properly. Use right side boundary condition $C(\tau, 10) = 8$. Compute your solution to 4 years before the expiry time.

(c). Vary your interest rate linearly from 0.04 to 0.14 from now (4 years before expiry date) to the expiry date. Vary your interest rate linearly from 0.14 to 0.04 from now to the expiry date.

(d). How would the volatility change the solution? Try $\sigma = 0.01$ and $\sigma = 0.5$.