Beacon based routing and coverage

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Abstract

We consider *beacon* based point-to-point routing and coverage problems motivated by sensor network applications. A beacon b is a point that can be *activated* to effect a 'gravitational pull' toward itself everywhere in a simply connected polygonal domain P.

We show that $\left\lfloor \frac{n}{2} \right\rfloor - 1$ beacons are sometimes necessary and always sufficient to route between any pair of points in P. We demonstrate that finding a minimum cardinality set of beacons to route from any source point $s \in P$ to a given destination $t \in P$ is NP-hard.

We show that it is NP-hard to find a minimum cardinality set of beacons to cover a simple polygon.

Keywords: path navigation, beacon, landmark, computational geometry, combinatorics, sensor networks

1 Introduction

The model of beacon based routing in this paper is an analog of geographical greedy routing in sensor networks in the continuous setting. In geographical routing [2, 5], each node is given a Euclidean coordinate and a message is delivered to the neighbor whose Euclidean distance to the destination is the smallest. When sensor distribution is very dense (i.e., close to infinity), geographical routing will always follow the straight line towards the destination, or, when the message hits the network boundary, may follow a boundary edge to greedily minimize the distance to the destination. This is precisely the model of beacon based routing in this paper, where the destination is a beacon.

Our model is also related to a family of routing schemes in sensor networks that use landmarks [3, 4, 7]. A subset of nodes, called landmarks, first flood the entire network such that each node records the distance to each landmark. For routing towards a destination, a function based on the distance vector to the landmarks is used to select the next hop. The one most similar to our model is adopted in [7], in which the message is routed towards one landmark until the current node is equal distance away from the landmark as the destination. At this point another landmark is selected. In this paper we examine the combinatorial structures for landmark placement, to support this type of routing.

The first problem we consider is that of finding a minimum cardinality set of beacons to route between two points in a simple polygon P (minimum beacon routing set). In our model, a beacon can occupy a point location on the interior or the boundary of P, ∂P. When a beacon is *activated*, all points $p \in P$ move along straight lines toward b until they either reach b or make contact with ∂P. If contact is made with ∂P, p will follow along ∂P as long as its straight line distance to b decreases monotonically. p may alternate between moving in a straight line path toward b on the interior of P and following along ∂P. If p is unable to move so that its distance to b decreases monotonically, we say p is 'stuck' and has reached a local minimum or *dead point* on ∂P (see Figure 1). If p reaches b we say that b *attracts* p. Two points are *routed* if there is a sequence of beacons that can be activated and then deactivated, one at a time in order, such that a point beginning at a source s would visit each beacon in the sequence after it is activated and terminate at a destination t, which we always assume to be a beacon itself.

2 Our Results

We first study the problem of finding a minimum cardinality set of beacons to route between two points in a polygon (minimum beacon routing set). We show that $\left\lfloor \frac{n}{2} \right\rfloor - 1$ beacons are sometimes necessary and always sufficient to route between any two points in a simple polyon P. We can see from Figure 2 that $\left\lfloor \frac{n}{2} \right\rfloor - 1$ beacons are sometimes necessary. To establish the upper bound, we consider the dual graph G of a triangulation of P and color the vertices of G blue or red. By appropriately placing a beacon in the cor-

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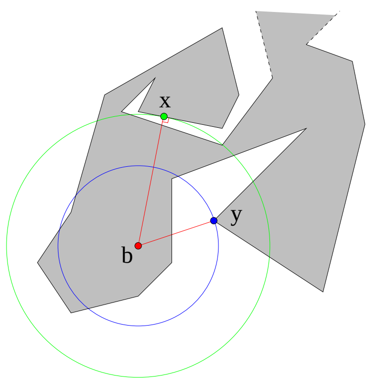

Figure 1: x and y are dead points with respect to the beacon, b.

responding triangle of each vertex of the lesser color class, we can guarantee the existence of a route between any pair of points in P (see [1] for details). Hence, we have:

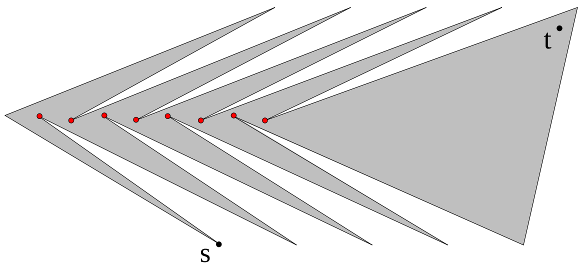

Figure 2: $\lfloor \frac{n}{2} \rfloor - 1$ beacons are sometimes necessary to route between all pairs of points in a simple polygon. Here, $n = 19$ and 8 beacons (light filled circles) are required to route from s to t.

Theorem 1. $\lfloor \frac{n}{2} \rfloor - 1$ *beacons are sometimes necessary and always sufficient to route between any pair of vertices in a simple polygon.*

In the subsequent theorem, we establish the hardness of all source routing in a simple polygon. The proof (omitted) is based on a reduction from the LINE HITTING problem.

Theorem 2. *It is NP-hard to find a minimum cardinality set of beacons to route from all source points s to a given destination point t in a simple polygon.*

The traditional art gallery problem is concerned with finding a small number of point guards to cover a polygon P on n vertices [8]. Finding a minimum sized guard set is known to be NP-hard [6]. Here, we establish the hardness of covering a simple polygon with beacons. We say P is covered by a beacon set B if for every $p \in P$, there exists a beacon $b \in B$ such that p is attracted to b. Again, the omitted hardness proof is based on a reduction from the LINE HITTING problem.

Theorem 3. *It is NP-hard to find a minimum cardinality set of beacons whose union covers a simple polygon.*

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