

AMS 151.1 Applied Calculus I

Practice Final Exam

Dec 17, 2007

Please write both your name and ID number on the answer sheet.

1. Simply the following two expressions as $m \ln(A) + n \ln(B)$ where m and n are constants.

(a) $\ln(AB) + \ln\left(\frac{B^2}{A}\right) + \ln(A^2B^2) + \ln(\sqrt{AB})$

(b) $\ln\left(A^{\frac{5}{3}}B^{\frac{2}{3}}\right) - \ln\left(\frac{1}{\sqrt{AB}}\right)$

2. Find both horizontal and vertical asymptotes for each of the following functions. If any of the asymptotes doesn't exist, clearly state that.

(a)

$$f(x) = \frac{2x + 1}{x - 2}$$

(b)

$$f(x) = \frac{36 - 4x^2}{x^2 - 2x + 1}$$

3. For what value of the constant c is the function f continuous where

$$f(x) = \begin{cases} cx^2 + 8 & \text{if } x \leq 3 \\ x^2 + cx + 2 & \text{if } x > 3 \end{cases}$$

4. For the following questions, find a formula for y' .

a) $y = \pi^2 + e^4 - 100$.

b) $y^3 = xy^2 + \ln(y) - 24$

c) $y = \sin^2\left(\frac{1}{x}\right) \cdot \ln(\sqrt{x})$

d) $y = \sin(\sin(\sin(x))) + \frac{\ln(x)}{\sin(x)}$

5. A hot air balloon is being decommissioned. Air is being removed at a rate of $10 \text{ ft}^3/\text{min}$. What is the rate of change of the radius of the balloon when it (the radius) is 2ft?

$$V = \frac{4}{3}\pi r^3$$

6. A 3m tall man throws a rock from the base of a building with initial velocity 12 m/s. Due to gravity, the height of the rock is given by the formula $h(t) = 4.9t^2 + 12t + 3$.

(a) What is the vertical speed of the rock 2 seconds into its flight?

(b) If a woman is gazing serenely out of a window 40m above the ground, does the rock fly high enough to hit her in the face? (explanation required)

7. The population of tribbles aboard the Klingon vessel Khu'vath doubles every hour(doubling time is 2). If the initial population of tribbles is 1:

a) Express the population of tribbles on the vessel as a function of time.

b) How many tribbles are on the vessel after the first 24 hours?

c) What is the rate of change of the tribble population after the first 24 hours?

8. A cone is being filled with lots of ice-cream. The base diameter equals to twice the height of the cone. If the influx of ice-cream is 10 gal/min, how fast is the water level rising when the volume is 5 gal.

$$\text{Remember: } V = \frac{1}{3}\pi r^2 h$$

9. The population of velociraptors in Jurassic park oscillates sinusoidally over time. The population maximum of 250 is reached on April. 1, and the population minimum of 100 raptors is reached on October. 1.

a) Find a sinusoidal function P which models the population of raptors as a function of t where t is in months.

b) What is the rate of change of the population on July 1?

10.

$$f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 28x + 79$$

a) On which intervals is this function increasing/decreasing?

b) On which intervals is f concave up/down

c) Identify any critical points for f .

d) Identify any inflection points for f .

11. Evaluate the following limits.

(a)

$$\lim_{x \rightarrow \infty} \frac{x^2 + 4x^4 - 21}{4x^3 - 12x + 5x^4}$$

(b)

$$\lim_{x \rightarrow 0} \frac{-2 \sin x + 4 \sin 2x}{\sin 4x}$$

(c)

$$\lim_{x \rightarrow \infty} 3x - 3xe^{\frac{1}{x}}$$