

AMS 151.1 Applied Calculus I

Final Exam

Dec 17, 2007

Please write both your name and ID number on the answer sheet.

1. Evaluate the following limits (using L'Hopital's rule if necessary).

(a)

$$\lim_{x \rightarrow \infty} \frac{x^4}{-1 + 2x^2 - x^4}$$

(b)

$$\lim_{x \rightarrow 1} \frac{\ln(x)}{x^2 - 1}$$

(c)

$$\lim_{x \rightarrow 0} 3x - 3e^x$$

2. Simply the following two expressions as $m \ln(A) + n \ln(B)$ where m and n are constants.

(a) $\ln(A^{\frac{5}{3}}) + \ln(\frac{1}{B^{\frac{2}{3}}}) - \ln(BA)$

(b) $\ln(\sqrt{AB}) + \ln(A^2B) + \ln(\frac{A}{B^2}) - \ln(A^2B^2)$

3. For what value of the constant c is the function f continuous where

$$f(x) = \begin{cases} cx^3 + 1 & \text{if } x \leq 1 \\ x^2 + 2cx + 3 & \text{if } x > 1 \end{cases}$$

4. Find both horizontal and vertical asymptotes for each of the following functions. If any of the asymptotes doesn't exist, clearly state that.

(a)

$$f(x) = \frac{x + \frac{1}{2}}{2x^2 - 8}$$

(b)

$$f(x) = \frac{3x^2 - 2x + 4}{x^2 - x - 12}$$

5.

$$f(x) = 21 + 21x - 9x^2 - x^3$$

- On which intervals is this function increasing/decreasing?
- On which intervals is f concave up/down
- Identify any critical points for f .
- Identify any inflection points for f .

6. For the following questions, find a formula for y' .

- $y = x^\pi - 100$.
- $xy^2 + \ln(y) + x^2 = \sin(x - \pi) + \ln(xy) - 24$
- $y = \ln\left(\frac{1}{x}\right) \cdot \frac{e^{2x+1}}{x}$
- $y = \cos(\ln(x)) + 2^x + x^{100}$

7. A rocket is taking off. It's position $S(t)$ is given by:

$$S(t) = \frac{-1}{3}t^3 + \frac{5}{2}3t^2 + 14t + 1$$

- What is the vertical speed of the rock 2 seconds into its flight?
- At what time does the rocket achieve its maximum height?

8. You are filling a spherical balloon with water (so you can hurl it at an unsuspecting math professor). The faucet releases 50 cm^3 of water per second. As the balloon is being filled, how fast is its radius changing when the volume is $36\pi \text{ cm}^3$. Remember the formula for the volume of a sphere is given as:

$$V = \frac{4}{3}\pi r^3$$

(hint: you will need to find r when the volume is 36π from this formula, as well as $\frac{dr}{dt}$ from its derivative)

9. Scientific studies show that the probability of having a car accident grows exponentially with the concentration of alcohol in the blood. If the probability of having an accident with an alcohol level of 0 is 0.06. Studies show that when the blood alcohol level is 0.05 the probability of having an accident is 0.12. (the units here are percent for both variables).

a) Express the odds of having a car accident, P , as a function of blood alcohol level, b . (It may help to see that the odds double between the two given values).

b) As Joe Bimbo of $A\Sigma\Sigma$ does a keg stand, his blood alcohol content is increasing at a rate of 0.01 percent per second. How fast is his likelihood of having a car accident increasing when his blood alcohol level is 0.06?

10. Jane Payne of $\Delta Y M$ is drinking beer out of a conical funnel whose base diameter is 8 cm, and height is 12 cm. If Jane can drink $50 \text{ cm}^3/\text{sec}$ of beer, then how fast is the beer level in the cone falling when it is 5cm full?

Remember: The ratio of height and radius for a cone is constant, and

$$V = \frac{1}{3}\pi r^2 h$$

11. A ball is attached to the end of a spring. When I flick the ball with my finger, it begins to wobble up and down. Suppose the ball starts at the position $x = 0$, and reaches its maximum height $x = 2$ when $t = 1$ second, and its minimum height $x = -2$ when $t = 4$ seconds.

a) Find a sinusoidal function $x(t)$ which models the position of the ball as a function of t where t is in seconds.

b) What is the speed of the ball after 5 seconds?