Homework 4
Due Date: December 1st, 2008

1. Pg114, Ex1:

   Let f be defined for all real x, and suppose that

   $$|f(x) - f(y)| \leq (x - y)^2$$

   for all real x and y. Prove that f is constant.

2. Pg114, Ex2:

   Suppose $f'(x) > 0$ in (a, b). Prove that f is strictly increasing in (a, b), and let g be its inverse function. Prove that g is differentiable, and that

   $$g'(f(x)) = \frac{1}{f'(x)} \qquad (a < x < b)$$

3. Pg114, Ex3:

   Suppose g is a real function on R, with bounded derivative (say $|g'| \leq M$). Fix $\epsilon > 0$, and define $f(x) = x + \epsilon g(x)$. Prove that f is one-to-one if ϵ is small enough. (A set of admissible values of ϵ can be determined which depends only on M.)

4. Pg114, Ex4:

   If

   $$C_0 + \frac{C_1}{2} + \cdots + \frac{C_{n-1}}{n} + \frac{C_n}{n+1} = 0.$$

   where $C_0, \cdots, C_n$ are real constants, prove that the equation

   $$C_0 + C_1 x + \cdots + C_{n-1} x^{n-1} + C_n x^n = 0$$

   has at least one real root between 0 and 1.

5. Pg114, Ex6:

   Suppose

   (a) f is continuous for $x > geq 0$,

   (b) $f'(x)$ exists for $x > 0$,

   (c) $f(0) = 0$,

   (d) f' is monotonically increasing.

   Put

   $$g(x) = \frac{f(x)}{x} \qquad (x > 0)$$

   and prove that g is monotonically increasing.

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