What is Important in School Mathematics

Preface

Near the end of July 2004, the Park City Mathematics Institute, under the direction of Herbert Clemens, hosted two workshops on states' K-12 mathematics standards. Both workshops were supported by the National Science Foundation (NSF). The first, July 21-24, was organized by Johnny Lott and was a meeting of the Association of State Supervisors of Mathematics (ASSM), the National Council of Teachers of Mathematics (NCTM), and some research mathematicians with an interest in K-12 mathematics education. The second workshop, July 25-28, the Mathematics Standards Study Group (MSSG) organized by Roger Howe, was a group of 12 mathematicians, many of whom had just been to the first workshop. During the discussion at the end of the first workshop a representative of the ASSM stood and asked the mathematicians: “What is important?” This essay is an attempt by the MSSG to begin an answer to that question. Other papers in the MSSG proceedings give answers to this question in terms of appropriate problem sets for state tests and in terms of two specific topics, place value arithmetic and proportions.

I. Introduction

This working paper presents a set of five principles that the mathematicians of the MSSG believe can provide a sound framework for the design of school mathematics curriculum and standards. These principles are simple and concise but also are meant to be of practical value to committees that are revising state mathematics standards. We realize that these principles are an initial effort that is incomplete in several ways. On the other hand, our group of mathematicians, who held diverse viewpoints on school mathematics, found considerable consensus on what these starting principles should be.

The discussion of the principles includes suggestions about curricula based on these principles. Equally important, the discussion presents a rationale for why we believe that instruction based on these principles can make a major difference in how well U.S. students are prepared for high school and college mathematics. Following this discussion, we present some advice about revising curricula and standards in light of these principles.

We only had time during our meeting to scratch the surface in examining school mathematics curricula and standards. For this reason, we chose to focus on the elementary grades, although most of the principles are relevant to mathematics taught in all grades. We concentrated on how mathematical instruction should start because having students’ mathematical learning commence on the right path is so critical to all future mathematical learning in school and college.

The value of a mathematical education and the power of mathematics in the modern world arise from the cumulative nature of mathematics knowledge. A small collection of simple facts combined with appropriate theory is used to build layer upon layer upon layer of ever more sophisticated mathematical knowledge. The essence of mathematical learning is the process of understanding each new layer of knowledge and thoroughly mastering that knowledge in order to be able to understand the next layer. The principles presented here are designed to promote such mathematical learning.

Principles for School Mathematics

1. Whole number arithmetic and the place value system are the foundation for school mathematics with most other mathematical strands evolving from this foundation. This foundation should be the subject of most instruction in early grades.
2. In every grade, the mathematics curriculum needs to be carefully focused on a small number of topics. Most mathematics instruction should be devoted to developing deepening mastery of core topics through computation, problem-solving and logical reasoning.
3. Instruction should be mathematically rigorous in a grade-appropriate fashion. All terms should be defined with language that is mathematically accurate. Key theorems and formulas should be proved, whenever possible.
4. Disciplined, mathematical reasoning is one of the most important goals of a school education. Although it is difficult to assess on statewide tests, it must permeate all mathematical instruction.

5 Most students should be taught the mathematical knowledge and reasoning skills needed to succeed in college. Students planning for a Bachelor's degree in a quantitative discipline should take a more demanding mathematics track in high school which prepares them to start calculus when they enter college.

II. Discussion of Principles

1. Whole number arithmetic and the place value system are the foundation for school mathematics with most other mathematical strands evolving from this foundation. This foundation should be the subject of most instruction in early grades.

By mathematical strands, we mean major content components that span many grades of school mathematics, such as geometry or pre-algebra/algebra. Some standards documents seem open to the misinterpretation that mathematics is a forest of distinct, albeit interconnected, strands. School mathematics has a more unified organization with almost all strands evolving out of a foundation of whole number arithmetic and the place value system.

We believe that this foundation ought to be the subject of almost all mathematics instruction in early grades. The arithmetic of fractions and decimals grow naturally out of this foundation. Measurement-- in time, in money, in weight, and in physical dimensions (length, area, and volume)-- arises as an extension of counting and provides contexts in which to practice arithmetic while also learning needed knowledge for daily life. Problems involving money lay a foundation for decimals. Missing number problems, such as 21 + __ = 58 and "Four times what is 12", and practice with the distributive law, such as simplifying 37*42 + 63*42, set the stage for algebra. Basic counting and comparisons of quantities can use histograms and other data displays. Practice identifying halves and quarters of cut-up circles and rectangles can give an early start to understanding fractions. What we are saying here may seem obvious to practicing teachers, but many state standards have so many components that a carefully designed development of arithmetic gets lost or is diminished by competing, secondary strands.

Mastering addition, subtraction, multiplication and divisions facts needs to be an incremental, evolving process, which carefully extends previous knowledge and constantly lays a solid foundation for future knowledge. Informal multiplication can begin very early with counting by 2's, 3's, 4's or 5's. And simultaneously division can start with finding how many groupings of 2's, 3's, 4's or 5's can be made from a given pile of, say, sticks. Connecting multiplication with division is critical to developing a sound understanding of division; division is possibly the most important of the basic arithmetic processes since it leads to fractions and proportions, a topic which too many U.S. students have great trouble mastering. As arithmetic becomes more formal, it can still be introduced incrementally; for example, multiplication by 7 could be introduced as the sum of multiplying by 2 and multiplying by 5. In this way, if students forget what 7×6 is, they can quickly refresh their memories by computing $2 \times 6 + 5 \times 6$.

Mastery of the place value system evolves similarly, possibly starting with sums of pennies and dimes. In time, students need to become proficient at decomposing numbers by powers of 10, e.g., $435 = 4 \times 100 + 3 \times 10 + 5 \times 1$, and applying the associative, distributive and commutative laws to perform the multi-digit algorithms in terms of such decompositions for addition and subtraction and later for multiplication. Also, from an early age, students need to be developing an understanding of the algebraic structure underlying arithmetic, e.g., that subtraction is the inverse of addition and later that division is the inverse of multiplication.

Here we give only suggestive hints of how a K-12 curriculum built on a foundation of whole number arithmetic could begin. We mathematicians recognize that the pedagogical expertise for the detailed implementation of such a curriculum lies with school mathematics teachers and mathematical education faculty. Our expertise is in the overall structure of mathematical knowledge and how the core topics in this structure develop across the school grades and on into college. From this viewpoint, we cannot overemphasize the requirement of a firm foundation in arithmetic and the place value system, both as preparation for mastery of later school mathematics and as a model for the power of mathematical methods.

Multi-digit arithmetic algorithms are a quintessential example of how a powerful mathematical theory is constructed. From single-digit addition facts, one derives the facts for subtraction and multiplication, and from multiplication comes division. Thus, a methodology is developed to add, subtract, multiply or divide any two numbers. This theory extends naturally to the arithmetic of fractions and decimals. More complicated calculations in algebra and later in college mathematics all are done using further incremental extensions of these basic algorithms. For this reason we want to stress the importance of these algorithms for students as preparation for studying mathematics in high school and, for the majority, later in college.

The firm foundation in arithmetic we advocate involves a solid dose of drill to build accuracy, speed and confidence. With calculators, there is less need for drill with very large numbers than there was 50 years ago, but we believe that many state standards reflect a swing too far in the other direction, especially in downplaying pencil-and-paper arithmetic drill with multi-digit numbers. Students need practice adding a column of, say, half a dozen 3-digit numbers and multiplying or dividing a 4-digit number by a 2-digit number. See the problems chapter for thoughtful examples of more complex arithmetic calculations. The only role for calculators in this process is to check answers computed by hand. Arithmetic proficiency can be continually reinforced by its use in problem-solving and in developing new mathematical knowledge.

We close this discussion by noting that there are two other related strands that belong in the early grades, namely, geometry and measurement. Students need to learn to recognize simple geometric shapes and subsequently to construct other figures from, and decompose them into, simple shapes. They need to use their growing skills in numbers and fractions to measure geometric attributes such as length and area and to link arithmetic to geometry, e.g., the area model for multiplication.

2. In every grade, the mathematics curriculum needs to be carefully focussed on a small number of topics. Almost all mathematics instruction should be devoted to developing deeper mastery of core topics through computation, problem-solving and logical reasoning.

After the near total focus on whole number arithmetic and the place value system in early elementary grades, the second half of elementary school mathematics ought to focus on arithmetic with fractions and decimals as well as the properties of these number systems. These number systems need to be understood in multiple ways. Students need to understand how to locate rational numbers on the (real) number line and to extend the number line to coordinates in the plane. Simple problems with proportions can be integrated into early calculations with fractions.

The development of arithmetic skills in integers, fractions and decimals should be matched with increasingly challenging applied problems, many in the context of measurement. In middle elementary grades, students can be solving simple two-step problems, such as: A store has the same price for all T-shirts. If three T-shirts cost $15, how much would 5 T-shirts cost? Towards the end of elementary grades, students can be solving multi-step problems, such as: If Sophie purchases a box of 24 apples for $4.50 and sells all the apples in packages of 3 apples with each package costing 85 cents, how much profit will she make?

Solving a problem in different ways ought to be an important aspect of mathematical reasoning with arithmetic and applied problems. (Alternative approaches should be mathematically substantive and fully understood by students; e.g., recasting multiplying a number by 98 as multiplying by (100-2) and using the distributive law.) A related skill is converting measurements between related units, e.g., between pounds and ounces, kilometers and meters, etc. Making up stories associated with arithmetic calculations is very helpful in firming up students understanding and reasoning about arithmetic.

Along with a primary focus on arithmetic for fraction and decimal number systems, students in later elementary grades and early middle school also need to study geometry and pre-algebra. In these grades, geometry is closely related to measurement, as students examine in greater depth the quantitative attributes of geometric figures (length of side, perimeter, area, volume, and size of angles). Ever more challenging problem-solving with geometric quantities develops geometric reasoning and arithmetic skills as well as general problem-solving skills.

Algebraic models grow from 'missing number' problems, e.g., how many boxes of 4 pencils are needed to supply 24 pencils, to problems such as: if a train traveling at a constant speed takes 2 hours and 40 minutes to go from Town A to town B which are 160 miles apart, how long will it take to go from town C to town D which are 225 miles apart. Preparatory work for algebra builds on arithmetic skills, measurement skills, geometric knowledge and problem-solving skills.

In middle school, a major focus of instruction ought to be deepening mastery of fractions and decimals involving a range of increasingly sophisticated problems with proportions, ratios and rates. Here is an example of the type of problem that students should work up to: A cook bought some eggs. She used 1/2 of them to make tarts and 1/4 of the remainder to make a cake. She had 9 eggs left. How many eggs did she buy? (Note: since the U.S. aspires to world-class education standards, our students need to be able to work such problems in 5^{th} grade, as is the norm in East Asian countries.)

Notice that much of what is found in standards for the elementary grades is not mentioned here. This is not accidental. Our goal is for students to develop an in-depth mastery of the mathematical knowledge and reasoning in core topics as they tackle increasingly challenging problems. This is the surest path to success in high school and college mathematics. Much of what we find in state standards is not relevant to that goal and ought not take time away from foundational issues that are crucial to success.

There are two important assumptions about this focused, incremental approach. First, no topic should be introduced until students have the background knowledge and general maturity to study it in depth. Second, after a certain point in the curriculum, students are expected to have mastered a particular skill or concept. While mastery of this topic will be re-enforced through its use in future learning, explicit instruction on the topic will no longer be given.

Finally, we say a word about data analysis and probability. In keeping with our concern for in-depth learning, we would prefer to see a solid development of data analysis for, say, six weeks during one year in high school, rather than a two-week chapter of data analysis every year starting somewhere in the elementary grades. A senior-level AP statistics is, of course, an even better way to learn statistics well. In elementary grades, data analysis can be presented informally in the context of data collection for applied arithmetic problems. The same arguments apply to probability, which first arises informally in applications of fractions.

3. Instruction needs to be mathematically rigorous in a grade-appropriate fashion. All terms should be defined with language that is mathematically accurate. Key theorems and formulas ought to be proved, whenever possible.

Such rigor starts in early grades when, for example, ad hoc methods for counting multiples of different numbers are superseded by the precision of the single-digit multiplication tables and later the standard multi-digit algorithm for multiplication. At the time this algorithm is presented, students need familiarity with an (age-appropriate) definition for multiplication: for any two whole numbers, for example, 3 and 5, 3 times 5 is the total number of elements in 3 groups which each contain 5 elements. They also need to have used this definition to verify the distributive and commutative rules for multiplication and the effect of multiplying by 10. With this foundation, all aspects of the multi-digit multiplication algorithm can be rigorously understood.

An example of such rigorous instruction in algebra is the quadratic formula. Some algebra instruction today only presents factoring, which is usually applied to quadratic equations with integer roots. Students should be proficient at finding roots by completing the square. This is an important mathematical technique that is used repeatedly in college mathematics. The instructor next can generalize the method of completing the square to derive the quadratic formula for determining the real roots, when they exist, to any quadratic equation. Students then need to work problems that are best solved with the quadratic formula.

Likewise, it is important for major theorems in Euclidean geometry with accessible proofs to be proved. The Pythagorean Theorem belongs in this group. Students pursuing technical majors in college are expected to understand and extend similar derivations, and even more abstract ones. Proofs in Euclidean geometry are a vital warm-up for these students. The development of disciplined reasoning is a critical component of mathematics education, and we believe there is still merit in the traditional view that geometry proofs are an important way to develop disciplined reasoning.

This rigorous approach requires that the mathematical terms that students learn and build upon should be mathematically accurate. For example, unit fractions, such as 1/4, need to be defined when students start to add in fractional units, such as 1/4 +2/4. At this time, the definition needs to be concrete, based on splitting a whole into equal-sized pieces; e.g., if a whole object, such as a square, is divided into four equal pieces, each piece is defined to be 1/4 of the whole. Later, by middle school, a unit fraction is defined more rigorously : for each positive integer n , the unit fraction 1/n is the multiplicative reciprocal of n. The earlier definition is 'upward compatible' with the later definition.

4. Disciplined, mathematical reasoning is one of the most important goals of a school education. Although it is difficult to assess on standardized tests, it must permeate all mathematical instruction.

The reasoning that mathematics develops is a basic life skill, as useful as arithmetic but harder to learn. It is valuable everywhere, in quantitative settings and in daily life. The ability to decipher statements and to analyze what they say and what they do not say will stand all students in good stead whatever their careers.

As more formal definitions are introduced, it is essential that students develop increasing skill at examining and understanding these definitions, through class discussions and exercises that force careful thinking about individual definitions and about connections among definitions. Understanding definitions and using them in mathematical arguments are at the heart of good mathematical reasoning.

The program of mathematics instruction described above with its focused, ever deepening development of mathematical knowledge, based on careful definitions, teaches students the mathematical reasoning and logical thinking that underlies all of mathematics. If this is done properly, students are constantly being led to extend their knowledge in a reasoned, incremental fashion to build layer upon layer of new knowledge. Students are constantly making connections among different pieces of knowledge. These processes of extending one's mathematical knowledge and making mathematical connections are the essence of mathematical reasoning and disciplined thinking. As an example, students need to be able to explain why the following definition is not satisfactory: A *cone* is a solid figure with a circular base and a curved surface that forms a point called the vertex.

In the later elementary grades, students can start to make general inferences about mathematical relationships and formulas. In high school, they will learn to construct simple proofs for themselves in Euclidean geometry.

Discipline mathematical reasoning requires mastery of basic logic, such as the relation between a statement, its inverse, its converse and its contrapositive. Constructing logical arguments has become a key component of all standards in light of the downgrading of extended proofs in school geometry. Formal justification of the elementary manipulations in algebra is a rich setting for logical reasoning; for example, the 'if and only if' nature of reversible algebraic operations. See Problem Set 3 for specific examples of such reasoning and related aspects of logical reasoning.

5 Most students need to be taught the mathematical knowledge and reasoning skills required to succeed in college-level mathematics. Students planning for Bachelor's degree in a quantitative discipline need a more demanding mathematics track in high school which prepares them to start calculus when they enter college.

Data from the U.S. Department of Education shows that today, about 75% of high school graduates enter two– or four-year college within two years of graduation. The earnings gap between college grads and those with only a high school diploma is continuing to grow. It follows that most students deserve a K-12 education in mathematics that prepares them to succeed in college-level mathematics.

The reality is that there are about 1,000,000 college students a year taking remedial courses covering high school mathematics that should have been taught and learned in courses in high school. Poor mathematical preparation is one of the leading factors in students' decisions to drop out of college. A good preparation for college-level mathematics comes from mathematics instruction that in every grade is rigorous, focused and continually challenging, in the spirit described in the preceding principles. In addition, we believe that all college-bound students should take mathematics every year in high school.

The students planning on quantitative majors need extensive instruction in trigonometry, logarithms, exponential functions, and analytic geometry. Again, poor mathematical preparation in these subjects is one of the leading reasons why college students drop out of technical majors like engineering. Our nation's heavy reliance of foreign-born graduate students in technical disciplines cannot be reduced without a conscious national effort to give future engineers and scientists a stronger mathematical and scientific education in secondary school.

Historically, there was a dramatic difference in the mathematics that college students in quantitative and non-quantitative majors needed to master. However, ever greater amounts of mathematics are now being required for more and more majors and 'white collar' professions. Labor economists such as Robert Reich have been documenting the growing gap between low-paying, low-skill service sector jobs and well-paying, high tech jobs that usually require strong quantitative skills. Thus, while students oriented towards quantitative majors need additional mathematics in high school, the difference in preparation is narrowing.

III. Advice for Revising School Mathematics Standards and Curriculum

The design of school mathematics standards and curriculum is a very complex, intellectually challenging task. We offer the following advice about this task.

A. States should seek out the best mathematical thinkers from schools, higher education and the private sector to serve on committees to design school mathematics standards and curriculum.

The outstanding credentials of members of such committees must reflect the intellectually challenging nature of designing of school mathematics standards and curricula. If mathematics education is to be given a high priority by states and they want expert guidance, then we believe that states would be well advised to follow the model used by the federal government, which turns to the National Academy of Sciences for expert advice. The Academy assembles panels of the nation's experts on a topic. These panels are chosen free of input from governmental officials or interest groups.

Such an expert panel for school mathematics would ideally be composed of distinguished scholars in mathematics and in mathematics education, along with representatives from the schools where the instruction occurs-- practicing teachers-- and representatives from companies and institutions who employ graduates-- mathematical experts from the private sector. The expertise of these groups is needed to design a focused, incremental curriculum, as outlined in the previous section, and to resolve conflicting objectives, e.g., simplicity and age-appropriateness versus mathematical correctness and completeness.

B. State mathematics programs have been redesigned too often. For help in developing more effective, stable mathematics programs, states are advised to draw heavily on successful mathematics programs in other countries, which have been gradually refined for many years.

All countries seek to teach their young people good mathematical skills and reasoning. It stands to reason that the experiences of other countries can be an important resource for U.S. standards developers. In virtually all commercial and intellectual activities, successful strategies incorporate the best ideas of others and then extend them. So it should be with school mathematics.

A number of East Asian countries have well documented track records of educating students who excel in mathematics at all levels in international comparisons, most recently in the Trends in International Mathematics and Science Study (TIMSS). These countries have school mathematics curricula that are widely judged to be effective and of high quality. We urge U.S. standards developers to use these successful East Asian mathematics curricula as a valuable resource in their work.

C. Greater precision and clarity are needed in the language in mathematics standards.

Terms like ‘reasoning’ and ‘understanding’ are used so extensively and in such general ways in many mathematics standards that they have lost meaning. Most standards documents use phrases that not only are vague but assume some context or additional information that is not explicitly stated. An example is the statement in one state’s standards about examining the “relationship between perimeter and area” of common geometric figures. The reality is that while some common geometric figures, e.g., a square, have such a relationship, many do not, e.g., an acute triangle. An example of mathematically incorrect reasoning appears in a standard directing students to use technology to show that rational numbers can be expressed as terminating or repeating decimals and irrational number as non-terminating and non-repeating decimals. While it is desirable to discuss the differences between rational and irrational numbers, this is a good example of a situation where technology is useless.

We believe that mathematicians have a natural role to play in polishing the language in standards, because they are experts in precise mathematical communication. For example, they could reformulate the flawed standards language mentioned in the previous paragraph to accurately communicate the intent of the standards writers. Also see A. above for our advice about an expert panel including mathematicians to design mathematics curriculum and standards.

D. Mathematics should arise in instruction in other school subjects in order to reinforce and apply learning in mathematics classes.

Proficiency in mathematics now ranks at the top of educational priorities along with proficiency in reading and writing. Reading and writing are developed in instruction in an array of different subjects. The same needs to be true for mathematics. Mathematical reasoning and problem-solving should be an integral component of school instruction in the sciences, in the same way that reading and writing are. In addition, work with displaying and interpreting graphical data should be part of social science instruction.

This recommendation applies to instruction in elementary school as well as middle school and high school. In elementary grades there is usually one teacher for all subjects, who in theory can integrate mathematics across the curriculum.

Appendix Piece

Historical Forces Shaping Today's School Mathematics Instruction
by Alan Tucker

It is natural to ask why a country with such scientific prowess has high school graduates who rank near the bottom in international comparisons of mathematical knowledge. This background section provides an answer.

Commercial thinking, which values numbers and accurate recording keeping, has shaped American life since colonial days. Colonial America had the world's first widespread public education, because everyone had to be prepared to work in a store or other small business, which required basic skill in reading, writing and arithmetic. The English chronicler Thomas Hamilton wrote in his 1833 visit to America, "Arithmetic I presume comes by instinct among this guessing, reckoning, expecting and calculating people." [Cohen, 1999] Today's businesses, run with spreadsheets, sophisticated forecasting models and statistical quality control, continue this tradition.

Quantitative intelligence, then, would seem to be a national strength. Yet until recently, only the most basic mathematical skills were needed for most people. (Even today, the No Child Left Behind Act addresses only minimal standards for all students; it sets no standards even for college-bound students.) The country's first colleges, created to train ministers, taught no mathematics or science. There was no training for teachers and theirs was one of the lowest ranked professions in early America.

Some founding fathers argued that a voting citizenry needed a deeper education. For example, George Washington wrote, "The science of figures. . . is not only indispensably requisite in every walk of civilized life, but the investigation of mathematical truths accustoms the mind to correctness in reasoning." However, Washington's type of education was associated with landed aristocracy, whose learning and power in Europe most immigrants to the U.S. despised. Further, it was deemed of little value in business. Our country quickly developed a tradition of anti-intellectualism in parallel to its support of basic education for all citizens.

In the late 1800's, there developed a reaction to excesses of the business world, especially for children. Along with brutal conditions in the workplace, reformers took aim at conditions in schools where knowledge was often beaten into children using the same tactics employed to train horses. The resulting progressive education movement was built around child psychology and the new 'science' of education that downplayed traditional academic subjects. While other countries were developing academically demanding goals in the early 1900's for high school education, focused on either vocation training or college preparation, U.S. high schools had vague educational goals. To many reformers, they were foremost semi-custodial institutions to keep young people out of dangerous factories. The NCTM was formed around 1920 to fight efforts to eliminate any mathematics course as a requirement for high school graduation. For more information about the history of U.S school education, see [Cohen, 1999 and Hofstadter, 1963] (the primary sources for this section).

The new science of education was matched by the new science of management, whose most famous proponent was Frederick Taylor. His 1911 book, The Principles of Scientific Management, proposed quantifying all aspects of the manufacturing process. He broke assembly line tasks down into minute steps. He even 'scientifically' determined the optimal physical measurements (e.g., height, girth of biceps) of a worker at each position on an assembly line. The size of management payrolls in companies exploded as a bureaucracy evolved to make sure all aspects of manufacturing were performed in a numerically optimal fashion. A contemporary version of Taylor's philosophy is the assertion: If you cannot measure it, you cannot manage it. Taylor's approach to management spread throughout business and other American institutions, including education. The current movement for school standards and annual tests is built firmly on Taylor's legacy.

But what are the mathematical goals of a school education? The American dream for business success is based more on entrepeneurism than technical knowledge. Most Americans today believe that more mathematics than whole number arithmetic is needed for good jobs in the future and that schools need to do a better job of preparing students for these jobs. Businesses today do not speak with one voice about what mathematics the workforce needs: Walmart and McDonald's require less mathematics of most employees than stores in colonial times-- no arithmetic beyond making change-- while GE and Hewlett-Packard require a lot more.

Mathematicians became deeply engaged in defining goals for school mathematics during the Cold War after the launch of Sputnik. A number of the country's leading mathematicians ran summer workshops for mathematics teachers in the 1960's. However, the 'New Math' movement over time became characterized by excess formalism that turned off the public as well as many mathematicians. Following the New Math debacle, the academic mathematics community withdrew from schools.

NCTM's 1989 Standards presented an innovative framework for school mathematics and more importantly launched a vast national movement for standards in school education. However, the Standards consisted general guidelines that were open to a wide range of interpretation and provoked considerable controversy. As a result, state mathematics standards that claim to implement the NCTM Standards vary widely, and standards writers are often motivated foremost by state and federal legislation requiring that some set of standards be created as input to mandated tests. In most states, committees to make up such standards are composed overwhelmingly of school teachers and 'lay people. This reflects a belief among elected officials and state education departments that school mathematics standards are a relatively straightforward matter. As noted in the lead chapter, the authors of this report disagree strongly with this viewpoint. We believe that creating good school mathematics standards and associated curricula is an intellectually complicated project requiring the best mathematical minds.

Historical References:

Patricia Cohen, 1999, *A Calculating People,* Routledge Press, New York.

Robert Hofstadter, 1963, Anti-Intellectualism in American Life, Vintage Books, New York; particularly Part V: Education in a Democracy.