AMS 161 Final Exam A Prof. Tucker Fall 2008

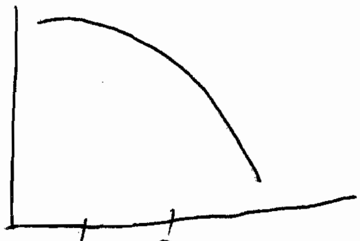
1. Do **three** of the following problems:

a) $\int_0^{\pi} [\sin(2x) - 10x^4]dx$, b) $\int 3x^3\sin(x^4)e^{2\cos(x^4)}dx$, c) $\int 4x/(x+1)dx$, d) $\int (2x-1)/\text{sqrt}(x + 1)\, dx$,

2. Do **two** of the following problems: a) $\int x\, e^{-2x}\, dx$, b) $\int \ln(3x)/x\, dx$, c) $\int x^9\cos(3x^5)\, dx$

3. Evaluate **two** of the following (explain answers): a) $\int_0^{\infty} 3t^7e^{-t^8}dt$, b) $\int_0^5 1/(x-3)^{2/3}dx$, c) $\int_4^{\infty} 1/(x-6)^2\, dx$.

4. Consider the integral from 0 to 2 of the function sketched at the right. List the values in INCREASING ORDER of the integral estimates given by the LH = Left-Hand Rule, RH = Right-Hand Rule, MP = Mid-Point Rule, and TP =Trapezoidal Rule in increasing order. Also indicate the position in this ordering of the true integral.



5. Do **one** of the following problems (JUST SET UP THE INTEGRAL):

a) Set up an integral for the volume of revolution around the y-axis (NOT x-axis) of $y = 3(x^2+5)^{1/2}$ from y=0 to y=2.

b) Set up an integral for the volume resulting from revolving the area between the curves y = 5+x and y = $1 + x^2$ between 1 and 2 about the line $y = 12$.

6. Do **two** of the following three problems: JUST SET UP THE INTEGRAL:

a) A thick rope holding a bucket of old math textbooks, weighing 50 lbs., hangs from the top of a 40-foot high building. The rope weighs 5 lbs. per foot. Set up an integral for the work to raise the bucket from ground level up to 30 feet above the ground.

b) A 150-foot high dam is shaped by the function $y = e^{2x2}$ -1 The water level is at the top of the dam. Set up an integral that gives the total water pressure against the dam. Water weights 62.4 lbs per cubic foot.

c) Water is being pumped into cone, with base at the bottom, that is 12 feet deep with a radius [redacted] 8 feet. *Set up an integral* for the work required to pump water up from ground level to fill the cone. Water weights 62.4 pounds per cubic feet.

7. a) Determine by direct computation the terms up to x^3 in the Taylor series for $1/(1-x)^2$. Show f',f'',f'''.

b) From part a), determine the terms up to x^6 in the Taylor series for $1/(1 - 2x^2)^2$.

c) Determine the terms up to x^4 in the Taylor series for $\sin(x)/(1 - 2x^2)^2$, where $\sin(x) = x - x^3/3! + x^5/5! - x^7/7!$

8. Determine the radius of convergence of the series $1 + 5x/1! + 5^2x^2/2! + 5^3x^3/3! + 5^4x^4/4! + 5^5x^55! + .$

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9. Solve **both** DEs: a) $y' = 2ye^{-x}, y(0) = 2$. b) $y'' - 8y' + 15y = 0,\ y(0) = 3, y'(0) = 2$.

10. Set up a Diff. Eqn. and solve it with given conditions for BOTH of the following two problems.

a) Newton's Law of Heating says that the rate at which the temperature of a cool object warms up to room temperature is proportional to the temperature difference between the object and the room. Hot coffee in a cup comes out of the micrwave oven at 150 ° F. into a room at 70° F. and in 10 minutes the coffee is 110° F. Find the temperature of the coffee in the cup as a function of time (in minutes) since coming out of the refrigerator.

b.) A reservoir holds 3,000,000 gallons of water. PCPs have started polluting the water, flowing into the reservoir at a concentration of .003 ounces per gallon of water. Each day 150,000 gallons of polluted flow into the reservoir and 150,000 gallons flow out of the reservoir into a nearby town's drinking water. Initially the reservoir has no PCPs. Set up and solve a differential equation for y(t), the amount of PCPs (in ounces) in the reservoir as a function of time (in days).