AMS 161 Final Exam A Prof. Tucker Spring 2008

18 pts, - 6 pt each

1. Do **three** of the following problems:

a) $\int_0^{\pi} [\cos(3x) - 6x]dx$, b) $\int x^2 \sin(e^{3x^3}) e^{3x^3} dx$, c) $\int 2x/(x-3)dx$, dx, d) $\int (3x+1)/\text{sqrt}(x+3)\, dx$,

16 pts - 8 pt each

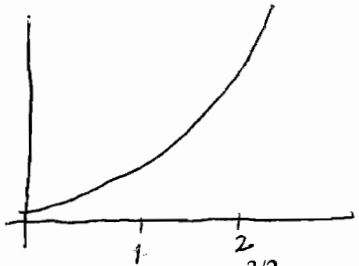
2. Do **two** of the following problems: a) $\int x \cos(3x)\, dx$, b) $\int \ln(2x)/x^4\, dx$, c) $\int x^{11} e^{-3x^6} dx$

12 pts - 6 pt each

3. Evaluate **two** of the following (explain answers): a) $\int_0^{\infty} 3t^7 e^{-t^8} dt$, b) $\int_0^5 1/(x-2)^{1/2} dx$, c) $\int_2^{\infty} 1/(x+3)^3\, dx$.

8 pt

4. Consider the integral from 0 to 2 of the function sketched at the right. List the values in INCREASING ORDER of the integral estimates given by the LH = Left-Hand Rule, RH = Right-Hand Rule, MP = Mid-Point Rule, and TP =Trapezoidal Rule in increasing order. Also indicate the position in this ordering of the true integral.



10 pt

5. Do **one** of the following problems (JUST SET UP THE INTEGRAL):

a) Set up an integral for the volume of revolution around the y-axis (NOT x-axis) of $y = 2(x+3)^{3/2}$ from y=0 to y=2.

b) Set up an integral for the volume resulting from revolving the area between the curves y = 10-x and y = x^2-1 between 1 and 2 about the line y = -6.

8 pt

6. Do **two** of the following three problems: JUST SET UP THE INTEGRAL:

a) A thick rope holding a bucket of unused calendars, weighing 30 lbs., hangs from the top of a 40-foot high building. The rope weighs 4 lbs. per foot. Set up an integral for the work to raise the bucket of from ground level up to 20 feet above the ground.

12 pt

b) A 200-foot high dam is shaped by the symmetric function y = sqrt (2x+1) -1 The water level is at the top of the dam. Set up an integral that gives the total water pressure against the dam. Water weights 62.4 lbs per cubic foot.

12 pt

c) Water is being pumped into trough that is 15 long, 5 feet dept and has V-shaped ends that are 8 feet across at the top (0 feet across at the bottom) *Set up an integral* for the work required to pump water up from ground level to fill the trough. Water weights 62.4 pounds per cubic feet.

9 pt

7. a) Determine by direct computation the terms up to x^3 in the Taylor series for $1/(x+1)^3$. Show f',f",f"'.

5 pt

b) From part a), determine the terms up to x^6 in the Taylor series for $1/(3x^2 + 1)^3$.

6 pt

c) Determine the terms up to x^4 in the Taylor series for $\cos(x)/(3x^2+1)^3$, where $\cos(x) = 1 - x^2/2! + x^4/4! - x^6/6!$

5 pt

8. Determine the radius of convergence of the series $1 + 3x/2^3 + 3^2x^2/2^3 + 3^3x^3/2^3 + 3^4x^4/2^3 + 3^5x^5/2^3 + .$

10 pt

9. Solve **both** DEs: a) $y' = 2xe^{-y}$, $y(0) = 0$.

12 pt

b) y" - 6y' + 8y = 0, y(0) = 2, y'(0) = 6.

10. Set up a Diff. Eqn. and solve it with given conditions for BOTH of the following two problems.

10 pt

a) Newton's Law of Heating says that the rate at which the temperature of a cool object warms up to room temperature is proportional to the temperature difference between the object and the room. Cold coffee in a cup comes out of the refrigerator at 50 ° F. into a room at 60° F. and in 10 minutes the coffee is 55° F. Find the temperature of the coffee in the cup as a function of time (in minutes) since coming out of the refrigerator.

15 pt

b.) A reservoir holds 4,000,000 gallons of water. PCPs have started polluting the water, flowing into the reservoir at a concentration of .002 ounces per gallon of water. Each day 200,000 gallons of polluted flow into the reservoir and 200,000 gallons flow out of the reservoir into a nearby town's drinking water. Initially the reservoir has no PCPs. Set up and solve a differential equation for y(t), the amount of PCPs (in ounces) in the reservoir as a function of time (in days).