AMS 161 Final Exam A Prof. Tucker Spring 2009

18 pts: 6 pts each

1. Do **three** of the following problems:

a) $\int_0^{\pi} [3x + \sin(4x)]\,dx$, c) $\int (1 + e^{\cos(x^2)})^2 x \sin(x^2) e^{\cos(x^2)}\,dx$, c) $\int 2x/(x-4)\,dx$, d) $\int (4x - 1)\text{sqrt}(x - 4)\,dx$,

16 pts: 8 pts each

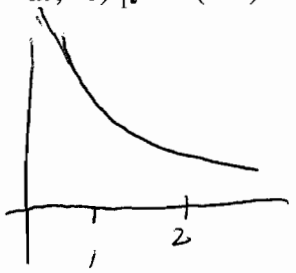
2. Do **two** of the following problems: a) $\int x\, e^{-4x}\,dx$, b) $\int x^2 \ln(6x)\,dx$, c) $\int x^7 \sin(3x^4)\,dx$

12 pts: 6 pts each

3. Evaluate **two** of the following (explain answers): a) $\int_0^3 1/x^{1/2}\,dx$, b) $\int_0^{\infty} t^5 e^{-t^6}\,dt$, c) $\int_1^{\infty} 1/(x-2)^{3/2}\,dx$.

8 pts

4. Consider the integral from 0 to 2 of the function sketched at the right. List the values in INCREASING ORDER of the integral estimates given by the LH = Left-Hand Rule, RH = Right-Hand Rule, MP = Mid-Point Rule, and TP =Trapezoidal Rule in increasing order. Also indicate the position in this ordering of the true integral.



10 pts

5. Do **one** of the following problems (JUST SET UP THE INTEGRAL):

a) Set up an integral for the volume of revolution around the y-axis (NOT x-axis) of $y = 2(x+1)^3$ from y=0 to y=2.

b) Set up an integral for the volume resulting from revolving the area between the curves $y = 2x+2$ and $y = x^2$ between 0 and 2 about the line $y = 10$.

6. Do **two** of the following (b) and C0 are worth more points): JUST SET UP THE INTEGRAL:

8 pts

a) A thick rope holding a bucket of calculus texts weighing 150 lbs. hangs from the top of a 50-foot high building. The rope weighs 3 lbs. per foot. Currently the bucket is 15 above the ground. *Set up an integral* for the work to raise the bucket of texts **up to the top** of the building.

12 pts

b) Water is being pumped into a conically shaped tank (pointed at the top). The tank's height is 40 ft. and its **radius** (at the bottom) is 25 ft. *Set up an integral* for the work required to pump water up from ground level to fill the tank. Water weights 62.4 pounds per cubic feet.

12 pts

c) A 100-foot high dam is shaped by the symmetric function $y = 3x^{4/3}$. The water level is at the top of the dam. Set up an integral that gives the total water pressure against the dam. Water weights 62.4 lbs per cubic foot.

11 pts

7. a) Determine by direct computation the terms up to x^3 in the Taylor series for $1/(x+3)$. Show f',f",f"'.

4 pts

b) From part a), determine the terms up to x^6 in the Taylor series for $1/(2x^3 + 3)$.

5 pts

c) Determine the terms up to x^6 in the Taylor series for $\sin(x)/(2x^3+3)$, where $\sin(x) = x - x^3/3! + x^5/5! - x^7/7!$

5 pts

8. Determine the radius of convergence of the series $1+2x/1+2^2x^2/2+2^3x^3/3+2^4x^4/4+$. SHOWYOUR WORK

10 pts

9. Solve **both** DEs: a) $y' = y/(3x)$, $y(1) = 3$; simplify answer (no log's)

12 pts

b) $y"-5y' + 4y = 0$, $y(0) = 3$, $y'(0) = 2$.

10. **Set up a Diff. Eqn. and solve it with given conditions for the following two problems.**

10 pts

a) Newton's Law of Heating says that the rate at which the temperature of a cool object warms up to room temperature is proportional to the temperature difference between the object and the room. Cold coffee in a cup comes out of the refrigerator at 35 ° F. into a room at 75° F. and in 12 minutes the coffee is 60° F. Find the temperature of the coffee in the cup as a function of time (in minutes) since coming out of the refrigerator.

15 pts

b.) A reservoir holds 3,000,000 gallons of pure water. DerivatumX starts polluting the water flowing into the reservoir at a concentration of .0025 pounds per gallon of water. Each day 200,000 gallons of polluted flow into the reservoir and 200,000 gallons flow out of the reservoir into a nearby town's drinking water. Initially the reservoir has no DerivatumX. Find y(t), the amount of DerivatumX in the reservoir as a function of time (in days).