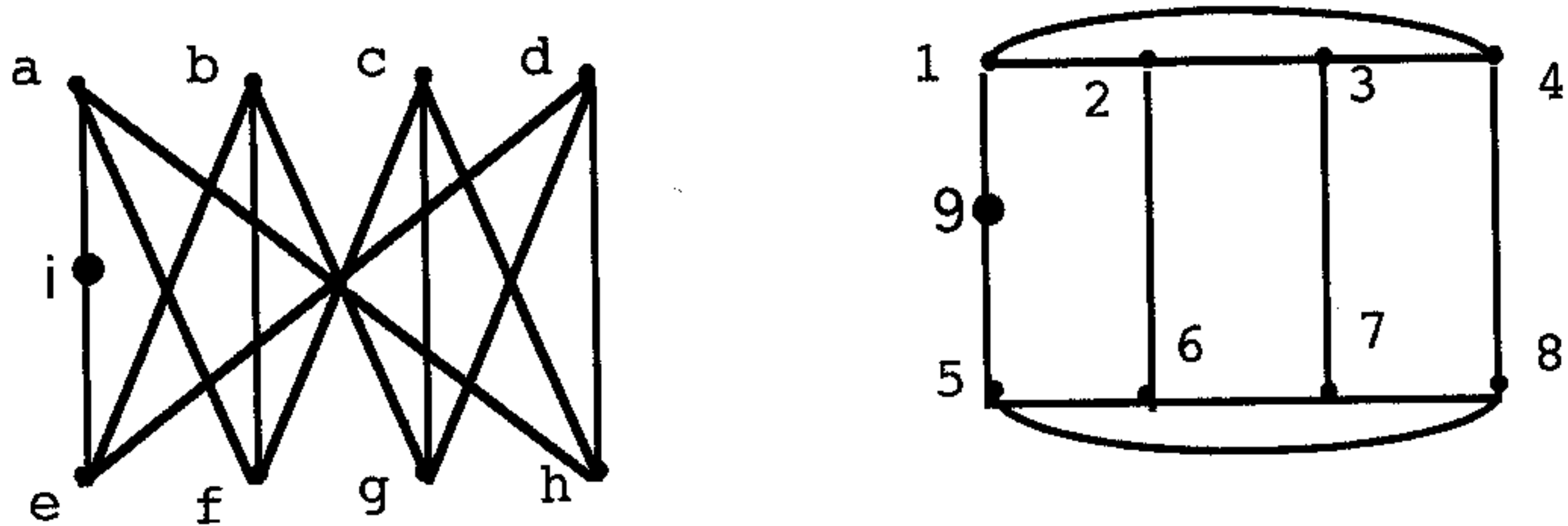


1. Are these two graphs isomorphic? Give the isomorphism or explain why none exists.



2. Find a lower bound on the minimal-cost traveling salesperson tour for the table on the right (using the method in the text). Suggest a good entry on which to branch. What is the new bound if you do not use this entry? If you do use this entry?

	1	2	3	4
1	$\infty$	9	3	7
2	7	$\infty$	7	3
3	5	6	$\infty$	4
4	7	8	5	$\infty$

	a	b	c	d	e	f	g
a	0	1	0	0	0	1	1
b	1	0	1	0	0	0	0
c	0	1	0	0	0	1	0
d	0	0	0	0	0	1	0
e	0	0	0	0	0	0	1
f	1	0	1	1	0	0	0
g	1	0	0	0	1	0	0

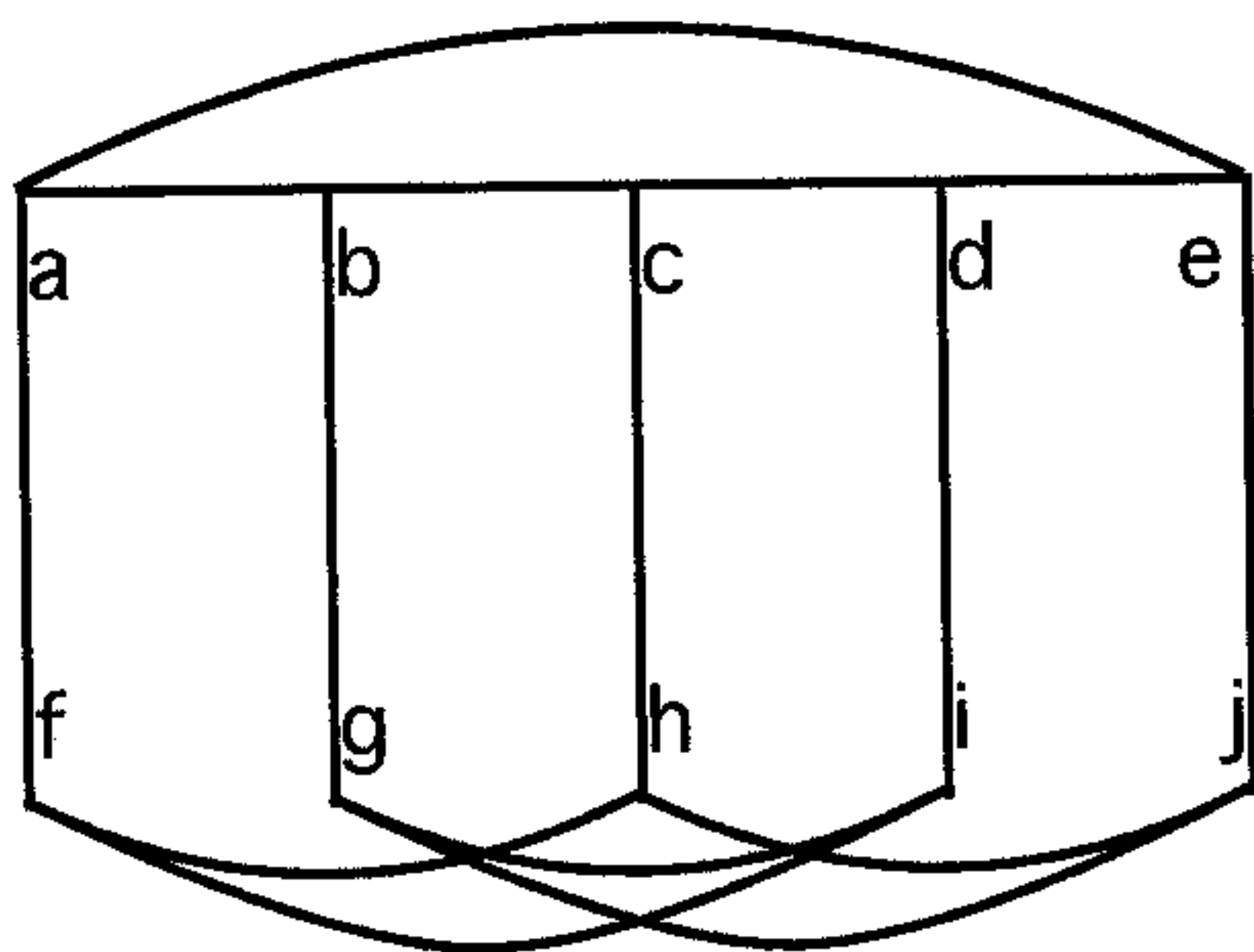
3. Is the graph with the adjacency matrix on the right connected? Test by trying to build a spanning tree found by a depth-first search starting at a.

4. A convention of actuaries will have rooms available in eight hotels. There are 200 actuaries and because of personality conflicts, various pairs of actuaries must be put in different hotels. The organizers wonder whether eight hotels will suffice to separate all conflicts. Model this conflict problem with a graph and restate the problem in terms of vertex coloring. What are the vertices? What are the edges? What are the colors?

5. Draw a connected planar graph (with no loops or multiple edges) for each of the following properties, if possible. If not possible, explain briefly why not.

- a) 13 edges and 9 regions (how many vertices must there be)
- b) 18 edges, and all vertices of degree 3; (how many vertices and regions must there be).
- c) has at least 5 vertices, has no Euler cycle and requires **exactly** 3 colors to properly color.

6. Give a careful argument to show that this graph has no Hamilton circuit.



7. A cutset in a connected graph  $G$  is a set  $K$  of edges whose removal disconnects  $G$ , but the removal of any proper subset of  $K$  does not disconnect  $G$ . For example, in the graph in problem 6, the set  $K = \{(a,e), (b,g), (c,h), (d,i), (e,j)\}$  is a cutset separating  $\{a,b,c,d,e\}$  from the rest of the vertices, while no proper subset of these 5 edges disconnects the graph.

- a) Find a cutset in the right graph in problem 1 with 4 edges.
- b) Explain briefly why any spanning tree of a connected graph  $G$  must intersect any cutset of  $G$ .
- c) If  $K$  is a cutset in a connected graph  $G$  and  $C$  is a circuit in  $G$ , explain briefly why  $K$  and  $C$  must have an even number of edges in common (possibly zero).