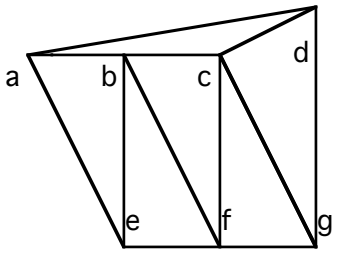
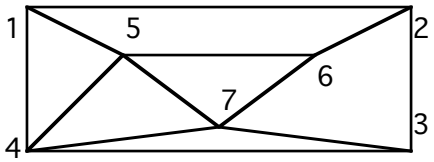
AMS 301.1 FIRST TEST - A Spring, 2008 Prof. Tucker

1. Are these 2 graphs isomorphic? Give the iso. or explain why none exists





2. Find a lower bound on the minimal-cost traveling salesperson tour for the table on the right (using the method in the text). Suggest a good entry on which to branch. What is the new bound if you do not use this entry? or if you do use this entry?

	1	2	3	4
1	∞	8	4	6
2	7	∞	4	5
3	6	4	∞	9
4	5	6	7	∞

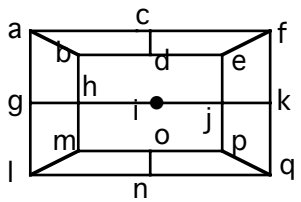
3. Is the graph with the adjacency matrix on the right connected? Test by trying to build a spanning tree found by a depth-first search starting at a.

	a	b	c	d	e	f	g
a	0	0	0	1	0	1	1
b	0	0	0	0	0	0	1
c	0	0	0	0	1	0	0
d	1	0	0	0	0	1	0
e	0	0	1	0	0	0	0
f	1	0	0	1	0	0	1
g	1	1	0	0	0	1	0

4. A zoo is going to place its animals in a set of large open areas, instead of having animals in individual cages. If two different animals cannot live together peacefully (e.g., a tiger and deer cannot live together because the tiger will eat the deer), then they must be put in different open areas. The zoo wants to determine the minimum number of open areas needed to safely house all its animals. Model this problem of assigning animals to a minimal number of open areas as a graph coloring problem. What are the vertices, the edges, the colors?

5. Draw a planar graph (with no loops or multiple edges) for each of the following properties, if possible. If not possible, explain briefly why not.
a) 7 vertices and 8 regions (how many edges must there be)
b) 10 edges, all vertices of degree 4 (how many vertices and regions must there be).
c) has exactly 8 vertices, has an Euler cycle and requires exactly 3 colors to properly color.

6. Give a careful argument to show that this graph has no Hamilton circuit



7. Consider a collection of circles (of varying sizes) in the plane. Make a 'circle graph' with a vertex for each circle and an edge between two vertices when they correspond to two circles that cross (if one circle properly contains another, there would be no edge).
a) Draw a family of circles whose circle graph is 5-chromatic (requires 5 colors to properly color).
b) Draw a family of circles whose circle graph has neither a Hamilton nor Euler circuit.
c) Draw a family of circles whose circle graph is a $K_{3,3}$.