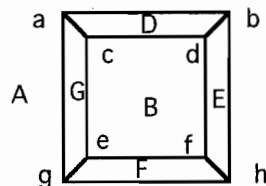
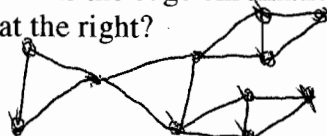


1a) What is the face chromatic number of the graph at the right?

b) What is the edge chromatic number of the graph at the right?



2. Find the chromatic polynomial for the graph ^{above} on the right.

3. Let G be a simple, connected plane graph. Explain the effect of the following operations on the number of vertices, edges and faces in G and in the dual of G .

- a) contract an edge; b) contract a triangular face.

4. Restate the following theorem in terms of the dual graph (DO NOT PROVE): If G is a simple connected planar graph with every vertex of degree at least four, then G has at least one triangular face.

5. Show if a planar graph G has all triangular faces and is vertex 3-colorable, then every vertex has even degree.

6. If T is a trail from vertex x to vertex y in a simple graph G (recall that a trail may repeat vertices but does not repeat edges), then prove that there exists a path P (with no vertices repeated) from x to y formed by a subset of vertices and edges of T .

7. Suppose that G is a connected, simple planar graph in which each vertex has degree 4.

a) Prove that $m = 2f - 4$.

b) Suppose that every face has 3 or 5 boundaries; that is, every face is triangular or pentagonal. Prove that there exists exactly 8 more triangular faces than pentagonal faces.