1. a) Show a minimal face coloring of this graph? Explain why fewer colors will not suffice.
   b) Show a minimal edge coloring of this graph? Explain why fewer colors will not suffice.

2. Re-state the following statement about a planar graph $G$ in the dual: If $G$ is a planar graph with every vertex of degree at least 4, then $G$ has at least two faces with bdy $\leq 5$. (JUST RESTATE FOR DUAL)

3. Find the chromatic polynomial of the following graph at the right.
4. What is the effect in the dual $G^*$ when the following operations are made in the planar graph $G$
   a) Delete a vertex of $G$ of degree 2   b) Contract a face of $G$ with 5 boundary edges.

5. Suppose that a graph $G$ has two different circuits $C$ and $C'$ that each contain the edge $e$. Show that $G$ must have a third circuit $C''$ that does not contain $e$. That is, explain how some of edges in $C$ and $C'$ can be used to construct a new circuit that does not contain $e$. Do not give an example but rather a general argument of how to construct such a $C''$. A general picture may help.

6. Show if a cubic (deg=3), planar graph $G$ is face 3-colorable, then every vertex has even degree in the dual. YOU CANNOT CITE ANY THEOREMS.

7. If $G$ is a simple, connected planar graph with all deg $\geq 3$ and less than 6 faces, prove that $G$ has a triangular face (bdy = 3). You may only assume Euler's formula $n-m+f=2$; prove everything else yourself.