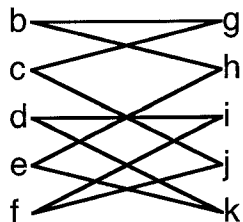


1. Give the cycle structure representation for a 160 degree rotation of the corners of a 9-gon.
2. a) Construct the matching network and make a flow corresponding to the partial matching b-g, c-j, d-i, e-k.  
 b) Apply the Augmenting Flow Algorithm (show ALL labels) and from it obtain a complete matching.



3. a) Consider the game of Nim on the right.  $||$   
 A player can remove any number of objects  $||||$   
 (exactly) one of the three piles. Find the  $|||||$   
 Grundy number for this position and  $||||||$   
 explain how to move to a position in the kernel.

- b) Explain how to move into a position with Grundy number of 2.
- c) Repeat part a) now with the constraint that a player must remove exactly 1 or exactly 4 objects from a pile.



4. 3. Give an expression for the pattern inventory of EDGE 2-colorings of this unoriented figure on the right (rotations and flips allowed).

5. Do one of the following two problems:

I. For vertices in a progressively finite graph, show that if the Grundy value  $g(x)$  of a vertex  $x$  is  $k$ , then there is a path of length at least  $k$  starting at  $x$ .

II. In the following table of remaining games, it is possible for the Vikings to be champions (or co-champions) if they win all remaining games? Build the appropriate network model and flow.

Team	Wins to date	Games to play	with Vikings	with Huns	with Romans	with Mongols
Vikings	22	6	—	2	2	2
Huns	27	6	2	—	2	2
Romans	26	6	2	2	—	2
Mongols	25	6	2	2	2	—