Spring 2004 AMS501: Homework #2
Due on Feb 16, 2004

1. 2.1.2 *Solution:* (a) Not separable. (b) $F(t,y) = yt^{-t}e^t$. (c) $F(t,y) = \sin y - t$. (d) Not separable.

2. 2.1.11 *Solution:* $y = Ce^{e^t}$, $y = e^{e^t-1}$, on $(-\infty, \infty)$.

3. 2.1.15 *Solution:* $y = \frac{1+ce^{t^2}}{1-ce^{t^2}}$, $c = -1$ on $(-\infty, \infty)$

4. 2.2.15 *Solution:* $m(x) = e^{x^2/2}$. $F(x,y) = e^{x^2/2}(y^2 + x)$.

5. 2.2.19 *Solution:* $m(y) = y^2$. $F(x,y) = x^2y^3 + y^4/2$.

6. 2.2.23 *Solution:* $m(x) = e^{\int p(x)dx}$.

7. 2.3.2 *Solution:*

8. 2.3.13 *Solution:*

9. 2.4.11 *Solution:* $y(t) \equiv 1$. $y = 1$ is a stationary point.

10. 2.4.22 *Solution:* (a) $K = 2$. (b) $K = 4$. (c) $K = 1$. (d) Not Lipschitz. (e) $K =$any positive constant

11. 2.4.23 *Solution:*

12. 2.5.4 *Solution:*

13. 2.5.11 *Solution:* $y = M$ is a stable solution and $y = 0$ is an unstable solution.

14. 2.5.18 *Solution:* $y = -1$ is unstable and $y = 4$ is stable.

15. 2.5.26. $f'(y_1) = 0$ should be $g'(y_1) = 0$. *Solution:*

16. Consider the differential equation $x' = 5(x-1)^{4/5}$.

    (a) Find the unique solution on the interval $[0,1]$ with $x(0) = 0$.

    Solution: $x(t) = (t-1)^5 + 1$.

    (b) For what values of α are there more than one solutions on $[0,\alpha]$ satisfying $x(0) = -31$. Find at least two solutions for such α.

    Solution: For $\alpha > 2$, there are infinitely many solutions. For any c between 2 and α, the following is a solution:

$$x(t) = \begin{cases} (t-2)^5 + 1 & \text{for } 0 \le t \le 2 \\ 1 & \text{for } 2 \le t \le c \\ (t-c)^5 + 1 & \text{for } c \le t \le \alpha \end{cases}$$

    (c) What is the reason that you have more than one solution in (b)?

    Solution: $(x-1)^{4/5}$ is not Lipschitz at $x = 1$.