AMS316 Exam 1 (2012)

1. Consider a series of observationas that are generated by the time series $X_t = a + bt + ct^2 + \epsilon_t$, where ϵ_t are independent and identically distributed Normal$(0, \sigma^2)$ random variables. One would decompose the series as $X_t = M_t + \eta_t$, in which M_t is the trend and η_t are noises. Use the moving average (i.e., smoothing) formula

$$Sm(X_t) = \frac{1}{2q+1} \sum_{r=-q}^{q} X_{t+r}$$

to get an estimate of the trend effect.

2. Suppose we have a seasonal seris of monthly observations $\{X_t\}$, for which the seasonal factor at time t is denoted by $\{S_t\}$. We further assume that the seasonal pattern is constant through time so that $S_t = S_{t-6}$ for all t. Denote a stationary seris of random variables by $\{\epsilon_t\}$. Consider the model $X_t = bt + S_t + \epsilon_t$ having a global linear trend and additive seasonality. Show that the seasonal difference operator $\nabla_6 = 1 - B^6$ acts on X_t to produce a stationary series.

3. Let B be the backward operator and X_t a time series, we know that $Bx_t = x_{t-1}$. Write the following time series as linear combinations of X_t's: (a) $(2 - B)(1 - B)X_t$, (b) $(B^3 - 2B^2 + 3B + 2)X_t$, and (c) $(1 - B^{12})X_t$?

4. Consider the series $X_t = 2t + t^3$. Compute its first-, second- and third-order differences ∇X_t, $\nabla^2 X_t$, and $\nabla^3 X_t$, where $\nabla = 1 - B$.

5. Given N observations $x_1, \ldots x_N$, on a time series. (a) Write down the estimation formula for **autocorrelation** coefficient at lag 1. (b) In general, what is your estimate for autocorrelation coefficient at lag k.

6. Suppose that a stationary stochastic process $X(t)$ have autocovariance function (acvf) $\gamma_X(\tau)$ and autocorrelation function (acf) $\rho_X(\tau)$. Show by the definition of acvf and acf that, $\gamma_X(\tau) = \gamma_X(-\tau)$ for $\tau = 1, 2, \ldots$ and $\rho_X(0) = 1$.

7. Write down the definition of the second-order (or weakly) stationary process.