1. Consider a series of observations that are generated by the time series \( X_t = a + bt + ct^2 + \epsilon_t \), where \( \epsilon_t \) are independent and identically distributed Normal(0, \( \sigma^2 \)) random variables. One would decompose the series as \( X_t = M_t + \eta_t \), in which \( M_t \) is the trend and \( \eta_t \) are noises. Use the moving average (i.e., smoothing) formula

\[
Sm(X_t) = \frac{1}{2q+1} \sum_{r=-q}^{q} X_{t+r}
\]

to get an estimate of the trend effect.

2. Suppose we have a seasonal series of monthly observations \( \{X_t\} \), for which the seasonal factor at time \( t \) is denoted by \( \{S_t\} \). We further assume that the seasonal pattern is constant through time so that \( S_t = S_{t-6} \) for all \( t \). Denote a stationary series of random variables by \( \{\epsilon_t\} \). Consider the model \( X_t = bt + S_t + \epsilon_t \) having a global linear trend and additive seasonality. Show that the seasonal difference operator \( \nabla_6 = 1 - B^6 \) acts on \( X_t \) to produce a stationary series.

3. Let \( B \) be the backward operator and \( X_t \) a time series, we know that \( Bx_t = x_{t-1} \). Write the following time series as linear combinations of \( X_t \)'s: (a) \((2 - B)(1 - B)X_t\), (b) \((B^3 - 2B^2 + 3B + 2)X_t\), and (c) \((1 - B^{12})X_t\).

4. Consider the series \( X_t = 2t + t^3 \). Compute its first-, second- and third-order differences \( \nabla X_t, \nabla^2 X_t, \) and \( \nabla^3 X_t \), where \( \nabla = 1 - B \).

5. Given \( N \) observations \( x_1, \ldots, x_N \), on a time series. (a) Write down the estimation formula for autocorrelation coefficient at lag 1. (b) In general, what is your estimate for autocorrelation coefficient at lag \( k \).

6. Suppose that a stationary stochastic process \( X(t) \) have autocovariance function (acvf) \( \gamma_X(\tau) \) and autocorrelation function (acf) \( \rho_X(\tau) \). Show by the definition of acvf and acf that, \( \gamma_X(\tau) = \gamma_X(-\tau) \) for \( \tau = 1, 2, \ldots \) and \( \rho_X(0) = 1 \).

7. Write down the definition of the second-order (or weakly) stationary process.