AMS316 Exam 2 (2012)

In the following problems, $\{Z_t\}$ is a purely random process, i.e., Z_t's are independent and identically distributed random variables with mean 0 and variance σ_Z^2.

1. Consider the random walk $S_t = S_{t-1} + Z_t$, is this process weakly stationary?

2. Consider the MA(2) process: $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}$. Compute its autocovariance function $\gamma_Z(\tau)$ and autocorrelation function $\rho_Z(\tau)$ for $\tau = 0, 1, 2, \ldots$.

3. Show that

   (a) the AR(2) process $X_t = -\frac{1}{12}X_{t-1} + \frac{1}{12}X_{t-2} + Z_t$ is stationary, and

   (b) the ACFs $\rho(k)$ $(k \geq 0)$ of the process are given by

$$\rho(k) = \frac{45}{77}\Big(-\frac{1}{3}\Big)^k + \frac{32}{77}\Big(\frac{1}{4}\Big)^k, \qquad \text{for } k \geq 0.$$

4. Consider the AR(2) process $X_t = \frac{1}{20}X_{t-1} + \frac{1}{20}X_{t-2} + Z_t$.

   (a) Show that the process is stationary.

   (b) Compute the ACFs $\rho(k)$ $(k \geq 0)$ of the process.