AMS316 Homework 2

2.1

The following data show the coded sales of company X in successive 4-week periods over 1995-1998.

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
1995	153	189	221	215	302	223	201	173	121	106	86	87	108
1996	133	177	241	228	283	255	238	164	128	108	87	74	95
1997	145	200	187	201	292	220	233	172	119	81	65	76	74
1998	111	170	243	178	248	202	163	139	120	96	95	53	94

(a) Plot the data.
(b) Assess the trend and seasonal effects.

2.2

Sixteen successive observations on a stationary time series are as follows: 1.6, 0.8, 1.2, 0.5, 0.9, 1.1, 1.1, 0.6, 1.5, 0.8, 0.9, 1.2, 0.5, 1.3, 0.8, 1.2
(a) Plot the data.
(b) Looking at the graph plotted in (a), guess an approximate value for the autocorrelation coefficient at lag 1.
(c) Plot x_t against X_{t+1}, and agian try to guess the value of r_1.
(d) Calculate r_1.

2.4

A computer generates a series of 400 observations that are supposed to be random. The first 10 sample autocorrelation of the series are $r_1 = 0.02,\ r_2 = 0.05,\ r_3 = -0.09,\ r_4 = 0.08,\ r_5 = -0.02,\ r_6 = 0.00,\ r_7 = 0.12,\ r_8 = 0.06,\ r_9 = 0.02,\ r_{10} = -0.08$. Is there any evidence of non-randomness?

2.5

Suppose we have a seasonal series of monthly observation $\{X_t\}$, for which the seasonal factor at time t is denoted by $\{S_t\}$. Further suppose that seasonal pattern is constant through time so that $S_t = S_{t-12}$ for all t. Denote a stationary series of random deviations by $\{\varepsilon_t\}$.
(a) Consider the model $X_t = a + bt + S_t + \varepsilon_t$ having a global linear trend and multiplicative seasonality. Show that the seasonal difference operator ∇_{12} acts on X_t to produce a stationary series.
(b) Consider the model $X_t = (a + bt)S_t + \varepsilon_t$ having a global linear trend and multiplicative seasonality. Does the operator ∇_{12} transform X_t to stationary? If not, define a differencing operator that does.
(Note: As stationary is not formally defined until Chapter 3, you should use heuristic arguments. A stationary process may involve a constant mean value (that could be zero) plus any linear combination of the stationary series $\{\varepsilon_t\}$, but should not include terms such as trend and seasonality.)