AMS 316 HW2 Solution

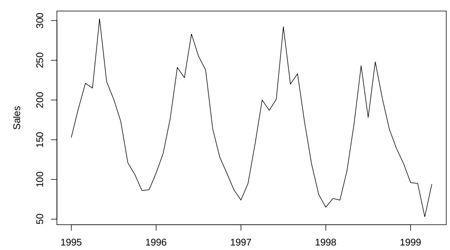
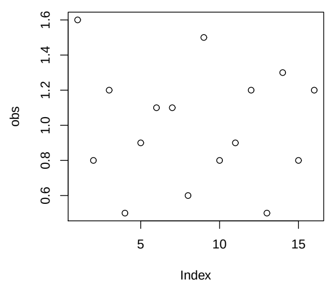


Figure 1: 1.(a)

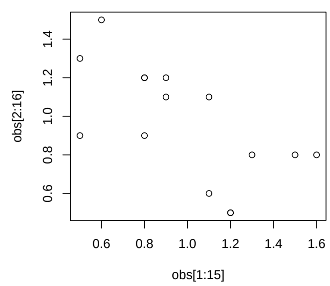
1. (a) R code:

```
data<-c(153, 189, 221, 215, 302, 223, 201, 173, 121, 106, 86, 87, 108,
        133, 177, 241, 228, 283, 255, 238, 164, 128, 108, 87, 74, 95,
        145, 200, 187, 201, 292, 220, 233, 172, 119, 81, 65, 76, 74,
        111, 170, 243, 178, 248, 202, 163, 139, 120, 96, 95, 53, 94)
data
dat<-ts(data, start=c(1995,1), frequency=13)
dat
plot(dat, xlab="", ylab="Sales")
```

   (b) The solution to this question depends on individual opinion. Students who argue for the existence of trend and seasonal effect can use various ways to assess them. A simple method is to calculate the four yearly averages in 1995, 1996, 1997 and 1998; and also the average sales in each of periods I,II,...,XIII(i.e. calculate the row and column averages). The yearly averages provide a crude estimates of trend, while the differences between the period averages and the overall average estimate the seasonal effects. With such a small downward trend, this rather crude procedure may well be adequate for most purposes. It has the advantage of being easy to understand and compute. A more sophisticated approach would be to calculate a 13-month simple moving average, moving along one period at a time. This will give trend values for each period from period 7 to period 46. The end values for periods 1 to 6 and 47 to 52 need to be found by some sort of extrapolation or by using a non-centered moving average. The difference between each observation and the corresponding trend value provide individual estimates of the seasonal effects. The average value of these differences in each of the 13 periods can then be found to estimate the overall seasonal effect, assuming that it is constant over the 4-year period. However, it's also reasonable to refuse to give any analysis if you think the time range is too short to detect any trend or seasonal effect.



(a) 2.(a)



(b) 2.(c)

2. (a) R code:

```
obs<-c(1.6, 0.8, 1.2, 0.5, 0.9, 1.1, 1.1, 0.6,
          1.5, 0.8, 0.9, 1.2,0.5, 1.3, 0.8, 1.2)
plot(obs)
```

   (b) From the graph, we can't get any reasonable guess on r_1.

   (c) R code:

```
plot(obs[1:15], obs[2:16])
```

   we can run a linear regression between obs[1:15] and obs[2:16] to guess r_1

```
lm.fit<-lm(obs[2:16]~obs[1:15])
summary(lm.fit)
lm.fit$coeff[2]
```

   (d) R code:

```
acf(obs)[1]
```

   $r_1 = -0.549$

3. The usual limits of 'significance' are at $\pm 2/\sqrt{N} = \pm 0.1$. Thus r_7 is just 'significant'. However, unless there is some contextual reason for an effect at lag 7, there is no real evidence of non-rannomness, as one expects 1 in 20 values to be 'significant' when data really are random.

4. (a)

$$\begin{aligned}\nabla_{12}X_t &= X_t - X_{t-12} \\ &= (a + bt + S_t + \epsilon_t) - (a + b(t-12) + S_{t-12} + \epsilon_{t-12}) \\ &= 12b + \epsilon_t - \epsilon_{t-12}\end{aligned}$$

Since $\{\epsilon_t\}$ is stationary series, ∇_{12} acts on X_t to prduce a stationary series.

(b)

$$\begin{aligned}\nabla_{12}X_t &= X_t - X_{t-12} \\ &= (a + bt)S_t + \epsilon_t - (a + bt - 12b)S_{t-12} - \epsilon_{t-12} \\ &= 12bS_{t-12} + \epsilon_t - \epsilon_{t-12} \\ &= 12bS_t + \epsilon_t - \epsilon_{t-12}\end{aligned}$$

Apparently, it's not a stationary series. Then we try ∇_{12}^2:

$$\begin{aligned}\nabla_{12}^2X_t &= (X_t - X_{t-12}) - (X_{t-12} - X_{t-24}) \\ &= 12bS_t + \epsilon_t - \epsilon_{t-12} - 12bS_{t-12} - \epsilon_{t-12} + \epsilon_{t-24} \\ &= \epsilon_t - 2\epsilon_{t-12} + \epsilon_{t-24}\end{aligned}$$

It's a linear combination of the stationary series ϵ_t. So the differencing operator ∇_{12}^2 is obtained.