AMS316 HW3

(Due Oct 24, 2011)

In the following equations, $\{Z_t\}$ is a discrete-time, purely random process such that $E(Z_t) = 0$, $\text{Var}(Z_t) = \sigma_Z^2$, and successive values of Z_t are independent so that $\text{Cov}(Z_t, Z_{t+k}) = 0$, $k \neq 0$.

3.1 Show that the ac.f of the second-order MA process

$$X_t = Z_t + 0.7Z_{t-1} - 0.2Z_{t-2}$$

is given by

$$\rho(k) = \begin{cases} 1, & k = 0 \\ 0.37, & k = \pm 1 \\ -0.13, & k = \pm 2 \\ 0, & \text{otherwise} \end{cases}$$

3.2 Consider the MA(m) process, with equal weights $\frac{1}{m+1}$ at all lags (so it is a real moving average), given by

$$X_t = \sum_{k=0}^{m} \frac{Z_{t-k}}{m+1}$$

Show that the ac.f of this process is

$$\rho(k) = \begin{cases} \frac{m+1-k}{m+1}, & k = 0, ..., m \\ 0 & k > m \end{cases}$$

3.3 Consider the infinite-order MA process X_t, defined by

$$X_t = Z_t + C(Z_{t-1} + Z_{t-2} + ...)$$

where C is a constant. Show that the process is non-stationary. Also show that the series of the first differences Y_t defined by

$$Y_t = X_t - X_{t-1}$$

is a first-order MA process and is stationary. Find the ac.f of Y_t .

3.5 If $X_t = \mu + Z_t + \beta Z_{t-1}$, where μ is a constant, show that the ac.f does not depend on μ .