

AMS316 HW4
(Due Nov 16, 2011)

1. Find the ac.f of the first-order AR process defined by

$$X_t - \mu = 0.7(X_{t-1} - \mu) + Z_t$$

Plot $\rho(k)$ for $k = -6, -5, \dots, 0, +1, \dots, +6$ (Remark: You can plot in Excel for convenience, if you want).

2. Assume the AR(2) process

$$X_t = \lambda_1 X_{t-1} + \lambda_2 X_{t-2} + Z_t$$

is stationary. If $\lambda_1 = 1/3, \lambda_2 = 2/9$, show that the ac.f of X_t is given by

$$\rho(k) = \frac{16}{21} \left(\frac{2}{3}\right)^{|k|} + \frac{5}{21} \left(-\frac{1}{3}\right)^{|k|}, k = 0, \pm 1, \pm 2, \dots$$

3. Show that the ac.f of the stationary second-order AR process

$$X_t = \frac{1}{12} X_{t-1} + \frac{1}{12} X_{t-2} + Z_t$$

is given by

$$\rho(k) = \frac{45}{77} \left(\frac{1}{3}\right)^{|k|} + \frac{32}{77} \left(-\frac{1}{4}\right)^{|k|} k = 0, \pm 1, \pm 2, \dots$$

4. For each of the following models:

(a) $X_t = 0.3X_{t-1} + Z_t$;

(b) $X_t = Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$;

(c) $X_t = 0.5X_{t-1} + Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$.

Express the model using B notation and determine whether the model is stationary and/or invertible.